

[Time: 3 Hours]

[Marks:80]

Please check whether you have got the right question paper.

- N.B:** 1. Q 1 is compulsory.
 2. Attempt any three from remaining
 3. Rights indicate full marks.

1. a. If A, B, C are subset of universal set V then prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$ **05**
 b. If $f: \mathbb{R} \rightarrow \mathbb{R}$ is given by $y = 2x + 1$, prove that f is one to one and onto and find f^{-1} **05**
 c. Find $L \{(1 + t e^t)^3\}$ **05**
 d. Check whether the following function Harmonic or not $3x^2 + \sin x + y^2 + 5y + 4$ **05**
2. a. Find k if $f(z) = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1} \frac{kx}{y}$ is analytic **06**
 b. Find $L \{|\sin 2t|\}$ **06**
 c. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ $f(x) = x^2 + 2x - 1$ **08**
 $g: \mathbb{R} \rightarrow \mathbb{R}$ $g(x) = 4x^2 + 2$
 Find (I) $f \circ g$ (II) $g \circ f$
3. a. Find Bilinear transformation under which $Z=1, -i, -1$ from point $w=i, 0, -i$ **06**
 b. If A be the set of non-integers and let R be a relation on $A \times A$ defined by $(a, b) R (c, d)$ if $ad=bc$, then prove that R is an equivalence relation. **06**
 c. Find (1) $L \left\{ \int_0^t e^u \frac{\sin u}{u} du \right\}$ **08**
 (2) $L \{(1 + 2t + 3t^2 + t^3)H(t - 2)\}$
4. a. Use convolution theorem and evaluate **06**

$$L^{-1} \left\{ \frac{(s+5)^2}{(s^2+10s+16)^2} \right\}$$

 b. Find transitive closure of following relation defined on $A = \{a, b, c, d, e\}$ by Warshall algorithm $R = \{(a, a) (a, b) (b, c) (c, d) (c, c) (d, e)\}$ **06**
 c. A man speaks truth 3 times out of 5 when a die is thrown he states that it gave an ace what is probability that this event has actually happened. **08**

5. a. How many four digit numbers can be formed out of the digits 1, 2, 3, 5, 7, 8, 9 if no digit is repeated twice? How many of them will be greater than 3000. **06**
- b. Solve using Laplace transform
 $\frac{d^2y}{dt^2} + 9y = 18$ given that $y(0) = 0$ and $y(\frac{\pi}{2}) = 0$ **06**
- c. Evaluate (1) $L^{-1} \left\{ \frac{1}{\sqrt{2s+1}} \right\}$ **08**
 (2) $L^{-1} \left\{ \frac{2s^2-6s+5}{s^3-6s^2+11s-6} \right\}$
6. a. Solve $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2, a_0 = 0, a_1 = 1$ **06**
- b. Find orthogonal curves of family of curves $e^{-x} \cos y + xy = \alpha$, where α is the real constant **06**
- c. i. Find the image of rectangular region bounded by $x=0, x=3, y=0, y=2$ under the transformation $w = z + (1+i)$ **08**
 ii. A fair dice is thrown thrice. Find probability that sum of numbers obtained is 10.
