Paper / Subject Code: 49801 / APPLIED MATHEMATICS- III

Time: 3 Hours **Marks: 80**

N.B.:1) Question no.1 is compulsory.
2) Attempt any three questions from Q.2to Q.6.
3) Figures to the right indicate full marks.

Q1. a) Find the Laplace transform of $e^{-t}t\cosh 2t$. **[5]**

b) Find the half-range cosine series for $f(x) = \begin{cases} 1 & ,o < x < \frac{a}{2} \\ -1 & ,\frac{a}{2} < x < a \end{cases}$ **[5]**

c) Find $\nabla\left(\bar{a}.\nabla\frac{1}{r}\right)$ where $\bar{a}$ is a constant vector. **[5]**

d) Show that the function $f(z) = z^3$ is analytic and find $f'(z)$ in terms of z. .**[5]**

Q2. a) Find the inverse Z-transform of $F(z) = \frac{3z^2-18z+26}{(z-2)(z-3)(z-4)}$,$3 < z < 4$. **[6]**

b) Find the analytic function whose imaginary part is $\tan^{-1}\left(\frac{y}{x}\right)$. **[6]**

c) Obtain Fourier series for the function $f(x) = \begin{cases} \frac{\pi}{2} + x & ,-\pi < x < 0 \\ \frac{\pi}{2} - x & ,0 < x < \pi \end{cases}$, **[8]**

Hence ,deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ and $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots$

Q3. a) Find $L^{-1}\left[\frac{s^2}{(s^2+1)(s^2+4)}\right]$ using convolution theorem. **[6]**

b) Show that the set of functions $\phi_n(x) = \sin\left(\frac{n\pi x}{l}\right), n = 1, 2, 3 \ldots$ is orthogonal in $[0\,, l]$. **[6]**

c) Using Green's theorem evaluate $\oint_C (e^{x^2} - xy)dx - (y^2 - ax)dy$ where C is the circle $x^2 + y^2 = a^2$. **[8]**

Q4. a) Find Laplace transform of $f(t) = \begin{cases} \frac{t}{a} & ,o < t \le a \\ \frac{(2a-t)}{a} & ,a < t < 2a \end{cases}$ and $f(t) = f(t + 2a)$. **[6]**

b) Prove that a vector field $\bar{f}$ is irrotational and hence find its scalar potential $\bar{f} = (y\sin z - \sin x)\,i + (x\sin z + 2yz)j + (xy\cos z + y^2)k$. **[6]**

c) Obtain the Fourier expansion of $f(x) = \left(\frac{\pi-x}{2}\right)^2$ in the interval $0 \le x \le 2\pi$ and $f(x + 2\pi) = f(x)$ .Also deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ **[8]**

Q5.a) Use Gauss's Divergence Theorem to evaluate $\iint_S \bar{N}.\bar{F}ds$ where $\bar{F} = 4xi + 3yj - 2zk$ and S is the surface bounded by x=0, y=0, z=0 and 2x+2y+z=4. **[6]**

b) Find the Z-transform of $f(k) = ke^{-ak}$, k≥ 0 . **[6]**

c) i) Find $L^{-1}\left[\frac{s+2}{s^2(s+3)}\right]$. **[8]**

ii) Find $L^{-1}\left[\log\left(\frac{s+a}{s+b}\right)\right]$.

Q6.a) Solve using Laplace transform $(D^2 + 3D + 2)y = 2(t^2 + t + 1)$, with $y(0) = 2$ and $y'(0) = 0$. **[6]**

b) Find the bilinear transformation which maps the points Z=1, i, -1 onto the points W=i, 0, -i . **[6]**

c) Find Fourier sine integral of $f(x) = \begin{cases} x & ,0 < x < 1 \\ 2 - x & ,1 < x < 2 \\ 0 & ,x > 2 \end{cases}$ **[8]**
