

(Time: 3 hours)

Max Marks: 80

Note: (1) Question No. 1 is Compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Figures to the right indicate full marks.

Q1.

- a) Solve $(y^2 e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)dy = 0$ 5
- b) Solve $(D^2 - 4D + 4)y = e^{2x} + \cos 2x$ 5
- c) Show that $\int_0^\infty \frac{e^{-x^3}}{\sqrt{x}} dx * \int_0^\infty y^4 e^{-y^6} dy = \frac{\pi}{9}$ 5
- d) Change the order of the following integration 5

$$I = \int_0^1 \int_{\sqrt{2x-x^2}}^{1+\sqrt{1-x^2}} f(x,y) dy dx$$

Q2.

- a) Evaluate $I = \iiint \frac{dx dy dz}{(x^2 + y^2 + z^2)^{3/2}}$, over the volume V bounded by the spheres $x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2$, $(b > a)$ 6
- b) Show that the length of the arc of the parabola $y^2 = 4ax$ from the vertex to the end of the latus rectum is $a[\sqrt{2} + \log(1 + \sqrt{2})]$. 6
- c) Solve by using method of variation of parameters 8

$$\frac{d^2 y}{dx^2} + y = \sec x \tan x$$

Q3.

- a) Show that $\int_0^\pi \frac{\log(1+a\cos x)}{\cos x} dx = \pi \sin^{-1} a, 0 \leq a \leq 1$. Hence 6
evaluate $\int_0^\pi \frac{\log(1+\cos x)}{\cos x} dx$

b) Evaluate $I = \iint y^2 dx dy$ over the area outside $x^2 + y^2 - ax = 0$ and inside $x^2 + y^2 - 2ax = 0$. 6

c) Evaluate $I = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} x^2 y z dx dy dz$ 8

Q4.

a) Solve $\cosh x \frac{dy}{dx} = 2 \cosh^2 x \sinh x - y \sinh x$ 6

b) Solve $\frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} + 2y = e^{e^x}$ 6

c) Show that $\int_0^\infty \frac{dx}{(e^x + e^{-x})^n} = \frac{1}{4} \beta\left(\frac{n}{2}, \frac{n}{2}\right)$, hence find the value of $\int_0^\infty \operatorname{sech}^6 x dx$ 8

Q5.

a) Evaluate $I = \int_{-1}^1 \int_0^{1-x} x^{1/3} y^{-1/2} \left((1-x-y)^{1/2}\right) dx dy$ 6

b) Find the area inside the circle $r = a$ and outside the cardioid $r = a(1 + \cos \theta)$ 6

c) Solve $xy(1 + xy^2) \frac{dy}{dx} = 1$ 8

Q6.

a) Solve $(D^2 + 2)y = x^2 e^{3x} + x \sin 3x$ 6

b) Solve $x e^x (dx - dy) + e^x dx + y e^y dy = 0$ 6

c) Change the order of integration and evaluate 8

$$I = \int_0^a \int_0^x \frac{dx dy}{(y+a)\sqrt{(a-x)(x-y)}}$$
