

Duration – 3 Hours

Total Marks: 80

- (1) **N.B.:-** Question no 1 is compulsory.
 (2) Attempt any **THREE** questions out of remaining **FIVE** questions.
 (3) **Figures to the right** indicate full marks.

Q.1.a) Solve $\left[y \left(1 + \frac{1}{x} \right) + \cos y \right] dx + (x + \log x - x \sin y) dy = 0$ (3)

b) Find the particular integral of $(D^2 - 2D + 1)y = xe^x \sin x$ (3)

c) Evaluate $I = \int_0^{\pi/4} (1 + \cos 4\theta)^5 d\theta$ (3)

d) Prove that $E \nabla = \nabla E$ (3)

e) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x dx dy dz$ (4)

f) Using Euler's method, find the approximate value of y, where $\frac{dy}{dx} = \frac{y-x}{\sqrt{xy}}$ (4)
 with $y(1) = 2$ when $x = 1.5$ in five steps taking $h = 0.1$

Q.2 a) Solve $dr + (2r \cot \theta + \sin 2\theta) d\theta = 0$ (6)

b) Evaluate $\int_0^{\infty} \frac{e^{-x}}{x} \left(1 - e^{-ax} \right) dx$ ($a > -1$) (6)

c) Change to polar and evaluate $I = \int_0^a \frac{\sqrt{a^2 - x^2}}{\sqrt{ax - x^2}} \frac{xdxdy}{\sqrt{(a^2 - x^2 - y^2)}}$ (8)
 (6)

Q.3 a) Evaluate $I = \int_0^1 x^4 \cos^{-1} x dx$

b) Evaluate $\iiint \frac{dxdydz}{x^2 + y^2 + z^2}$ throughout the volume of the sphere $x^2 + y^2 + z^2 = a^2$ (6)

c) Apply method of variation of parameter to solve $\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = e^{-x} \log x$ (8)

- Q. 4 a) Find the mass of a plate in the form of a cardioid $r = a(1 - \cos \theta)$, if the density at any point of the plate varies as its distance from the pole. (6)
- b) Solve $\frac{d^2 y}{dx^2} - 4\frac{dy}{dx} + 3y = 2xe^{3x} + 3e^x \cos 2x$ (6)
- c) Using fourth order Runge-Kutta method, solve numerically, the differential equation $\frac{dy}{dx} = xy$ with the given condition $y(1) = 2$, find y at $x = 1.2, 1.4$ (8)
- Q. 5 a) Evaluate $\iint xy \, dx \, dy$ over the region bounded by $x^2 + y^2 - 2x = 0$, $y^2 = 2x$ and $y = x$ (6)
- b) A resistance of $100 \, \Omega$ and inductance of $0.5 \, \text{H}$ are connected in series with a battery of $20 \, \text{V}$. Find the current at any instant if the relation between L , R , E is $L\frac{di}{dt} + Ri = E$ (6)
- c) Evaluate $\int_0^1 \frac{dx}{1+x}$ by using (i) Trapezoidal Rule, (ii) Simpson's $(1/3)^{\text{rd}}$ Rule and (iii) Simpson's $(3/8)^{\text{th}}$ Rule. Compare the result with exact solution. (8)
- Q. 6 a) Solve $(3x+2)^2 \frac{d^2 y}{dx^2} + 3(3x+2) \frac{dy}{dx} - 36y = 3x^2 + 4x + 1$ (6)
- b) Show that the length of the parabola $y^2 = 4ax$ from the vertex to the end of the latus rectum is $a\left[\sqrt{2} + \log(1 + \sqrt{2})\right]$ (6)
- c) Find the volume bounded by the paraboloid $x^2 + y^2 = az$ and the cylinder $x^2 + y^2 = a^2$ (8)