(Time: 3 Hours) **Max. Marks: 80**

N.B: 1) **Question No.1** is **COMPULSORY**

2) Answer **ANY THREE** questions from **Q. 2 to Q. 6**

3) Figures to the right indicate full marks

Q.1 a) Solve 5

$$(2x - 2x^3y^2)dy + (x^2y^3 + 2y)dx = 0$$

b) Show that 5

$$\int_0^{\pi/2} \sqrt{\sin\theta}\ d\theta * \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$$

c) Evaluate 5

$$\int_0^{\log 2} \int_0^{x} \int_0^{x-y} e^{x+y+z} dz\, dy\, dx$$

d) Solve 5

$$(D^2 + 3D + 2)y = \sin(e^x)$$

Q.2 a) Evaluate $\int_0^1 \frac{x^a - 1}{\log x}\, dx$ using DUIS rule 6

b) Change the order of Integration 6

$$\int_0^{a} \int_{\sqrt{a^2-x^2}}^{x+3a} f(x,y)\, dy\, dx$$

c) Solve by the method of variation of parameters 8

$$\frac{d^2y}{dx^2} + y = \sec x \tan x$$

Q.3 a) Evaluate $\iiint dx\, dy\, dz$ over the solid region of paraboloid $x^2 + y^2 = 4z$ cut off by the plane $z = 4$ 6

b) Solve 6

$$y^4\, dx = \left(\frac{1}{x^{3/4}} - xy^3\right) dy$$

c) Prove that 8

$$\left(\int_0^3 \frac{x^{3/2}}{\sqrt{3-x}}\, dx\right) * \left(\int_0^1 \frac{1}{\sqrt{1-x^{1/4}}}\, dx\right) = \frac{432}{35}\pi$$

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Q.4 a) Solve — 6

$$[y\sin(xy) + xy^2\cos(xy)]dx = -[x sin(xy) + x^2 y\cos(xy)]dy$$

b) Change the order of the integration and evaluate — 6

$$\int_0^1 \int_{-\sqrt{y}}^{-y^2} xy\,dx\,dy$$

c) Solve — 8

$$(D^2+2)y = e^x\cos x + x^2e^{3x}.$$

Q.5 a) Solve $x sin\, x\, dy + (x y cos\, x - y sin\, x - 2)dx = 0$ — 6

b) Evaluate $(D^2+1)y = 2^x + \sin x \sin 2x$ — 6

c) Using Polar co-ordinates, evaluate $\iint \frac{(x^2+y^2)^2}{x^2y^2}\,dx\,dy$ over the area common to the circles $x^2+y^2 = ax$ and $x^2+y^2 = by$,$a > 0, b > 0$ — 8

Q.6 a) Find the length of the cardioid $r = a\,(1-\cos\theta)$ lying outside the circle $r = \mathrm{a}\cos\theta$ — 6

b) Evaluate $\iint xy\,dx\,dy$ over the region bounded by the curves $y = 4x, x + y = 3, y = 0$ and $y = 2$ — 6

c) Evaluate $\iiint (x^2+y^2+z^2)\,dx\,dy\,dz$ over the positive octant of the sphere $x^2+y^2+z^2 = 4$ — 8
