Time : 3 Hours **Max Marks : 80**

N.B. : 1) Question number 1 is **compulsory**
2) Answer **any three questions** from the remaining questions Q2 to Q6.
3) Figures to the right indicate full marks

Q.1 a) Solve $\frac{dy}{dx}+\frac{4x}{x^2+1}y=\frac{1}{\left(x^2+1\right)^3}$ **(5)**

b) Evaluate $\int_0^{2a} x^2\sqrt{2ax-x^2}\,dx$ **(5)**

c) Solve $\left(D^3+3D^2+3D+1\right)y=e^{-x}$ **(5)**

d) Change to polar coordinates and evaluate $\int_0^a\int_y^a x\,dx\,dy$ **(5)**

Q.2 a) Solve $\frac{dy}{dx}=2+\sqrt{xy}$ **(6)**
with $x_0=1.2$, $y_0=1.6403$ by Euler's modified formula for $x=1.4$, $x=1.6$

b) Evaluate $\iint y\,dx\,dy$ throughout the area bounded **(6)**
by x=0, $y=x^2$ and $x+y=2$.

c) Solve $\left(4xy+3y^2-x\right)dx+x\left(x+2y\right)dy=0$ **(8)**

Q.3 a) Evaluate the integral $\int_0^2\int_1^2\int_0^{yz} x\,y\,z\,dx\,dy\,dz$ **(6)**

b) Assuming the validity of differentiation under the integral sign, **(6)**
Prove that $\int_0^1 \frac{x^a-x^b}{\log x}dx=\log\frac{a+1}{b+1}$

c) Solve $\left(D^3+D\right)y=\sin x$ **(8)**

Q.4 a) Solve $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=\frac{e^{-2x}}{x^5}$ **(6)**

b) Change the order of integration for the integral $\int_0^a\int_{y^2/a}^{y} f(x,y)\,dx\,dy$ **(6)**

c) Solve $\frac{d^2y}{dx^2}-y=\frac{2}{1+e^x}$, **(8)**
Using the method of variation of parameters

Q.5 a) Find $\int_0^1 \int_0^y x\,y\,e^{-x^2}\,dx\,dy$ **(6)**

b) Apply Runge-Kutta method of fourth order to find an approximate value of y at y=0.2 if $\frac{dy}{dx} = x + y^2$ given that y=1 when x=0 in steps of h=0.1 **(6)**

c) Solve $\frac{dy}{dx}\cosh x = 2(\cosh x)^2 \sinh x - y \sinh x$ **(8)**

Q.6 a) Evaluate $\int_0^3 e^{\sqrt{x}}\,dx$ by Simpson's $3/8^{th}$ rule. Take $h = 0.25$. **(6)**

b) Solve $\left(x^3y^3 + xy\right)dy = dx$ **(6)**

c) Evaluate $\int_0^2 y^4\left(8 - y^3\right)^{-1/3}\,dy$ **(8)**
