(3 Hours) [Total Marks : 80]

N.B 1) Question **No. 1** is **Compulsory**.

2) **Answer** any **three** questions from remaining questions.

3) Figures to the right indicate full marks.

Q.1 a) Evaluate $\int_0^\infty xe^{-x^4}\,dx$. 3

b) Find the length of the arc of the curve $r = asin^2\left(\frac{\theta}{2}\right)$ from $\theta = 0$ to any point $P(\theta)$. 3

c) Solve $(D^4 - 2D^2 + 1)y = 0$. 3

d) Solve $(x - 2e^y)dy + (y + xsinx)dx = 0$. 3

e) Evaluate $\int_0^1 \int_0^x x^2y^2(x + y)dydx$. 4

f) Solve $\frac{dy}{dx} = x^3 + y$ with initial condition $x_0 = 1,\ y_0 = 1$ by Taylors method. Find the approximate value of y for x=0.1. 4

Q.2 a) Solve $\frac{d^2y}{dx^2} - 4y = x^2e^{3x} + e^{3x} - sin2x$. 6

b) Show that $\int_0^\infty \frac{log(1+ax^2)}{x^2}dx = \pi\sqrt{a}$,$(a > 0)$ 6

c) Change the order of integration and evaluate $\int_0^5 \int_{2-x}^{x+2} dydx$. 8

Q.3 a) Evaluate $\iiint z\,dxdydz$ over the volume of tetrahedron bounded by the planes $x = 0, y = 0, z = 0\ and$ 6

$$\frac{x}{3} + \frac{y}{4} + \frac{z}{5} = 1.$$

b) Find the mass of the lamina bounded by the curves $y^2 = 4x$ and $x^2 = 4y$ if the density of the lamina at any point varies as the square of its distance from the origin. 6

c) Solve $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = -x^4sinx$. 8

Q.4 a) Find by the double integration the area between the curves $y^2 = 4x$ and $2x - 3y + 4 = 0$. 6

b) Solve $(1 + siny)\frac{dx}{dy} = 2ycosy - x(secy + tany)$. 6

c) Solve $\frac{dy}{dx} = x^2 + y^2$ with initial conditions $y_0 = 1$, $x_0 = 0$ at at x=0.2 in steps of h=0.1 by Runge Kutta method of fourth order. 8

Q.5 a) Evaluate $\int_0^1 \frac{x^2}{\sqrt{1-x^4}}dx \cdot \int_0^1 \frac{dx}{\sqrt{1-x^4}}$. 6

b) The distance x descended by a parachute satisfies the differential equation $\left(\frac{dx}{dt}\right)^2 = k^2\left(1 - e^{-2gx/k^2}\right)$where k and g are constants. If x=0 when t=0, show that $x = \frac{k^2}{g} logcosh\left(\frac{gt}{k}\right)$. 6

c) Evaluate $\int_0^1 \frac{dx}{1+x^2}$by using i) Trapezoidal ii) Simpsons (1/3)rd and iii) Simpsons (3/8)th rule. 8

Q.6 a) Find the volume in the first octant bounded by the cylinder $x^2 + y^2 = 2$ and the planes $z = x + y, y = x$, $z = 0 \; and \; x = 0$. 6

b) Change to polar coordinates and evaluate $\iint_R \frac{dxdy}{(1+x^2+y^2)^2}$ over one loop of the lemniscates $(x^2 + y^2)^2 = x^2 - y^2$. 6

c) Solve by method of variation of parameters $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$. 8
