

Duration – 3 Hours

Total Marks: 80

- N.B.** 1. Question No. 1 is compulsory.  
 2. Attempt any **THREE** questions out of remaining **FIVE** questions.  
 3. Figures to right indicate full marks.

1) a) Solve  $2(x^2\sqrt{y} + 1)y dx + (x^2\sqrt{y} + 2)x dy = 0$  (4)

b) Find the particular integral of  $(D-3)y = x$  (3)

c) Evaluate  $\int_0^{\infty} e^{-x^2} dx$  (3)

d) Sketch the region of integration  $I = \int_1^4 \int_0^{\sqrt{x}} \frac{3}{2} e^{(y/\sqrt{x})} dy dx$  (3)

e) Prove that  $E = 1 + \Delta = e^{hD}$  (3)

f) Using Euler's method find the approximate value of y, where  $\frac{dy}{dx} = \frac{y-x}{\sqrt{xy}}$  (4)

and  $y(1) = 2$  when  $x = 1.5$  in five steps taking  $h=0.1$

2 a) Solve  $\frac{dy}{dx} + y = y^2(\cos x - \sin x)$  (6)

b) Show that  $\int_0^{\infty} \frac{\tan^{-1} ax}{x(1+x^2)} dx = \frac{\pi}{2} \log(1+a)$ . Hence evaluate  $\int_0^{\infty} \frac{\tan^{-1} x}{x(1+x^2)} dx$  (6)

c) Change to polar and evaluate  $I = \int_0^a \int_y^{a+\sqrt{a^2-y^2}} \frac{dx dy}{(4a^2 + x^2 + y^2)^2}$  (8)

3 a) Given that  $\int_0^{\infty} \frac{x^{p-1}}{1+x} dx = \frac{\pi}{\sin p\pi}$  .P.T  $\Gamma(p)\Gamma(1-p) = \frac{\pi}{\sin p\pi}$  ( $0 < p < 1$ ) (6)

b) Evaluate  $\iiint_V \frac{dx dy dz}{(1+x^2+y^2+z^2)^2}$  where V is the volume in the first octant. (6)

c) Solve by method of variation of parameters  $\frac{d^2 y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$  (8)

- 4 a) Evaluate  $I = \int_0^{\pi} 2d\theta \int_0^{a(1+\cos\theta)} r dr \int_0^h \left[ 1 - \frac{r}{a(1+\cos\theta)} \right] dz$  (6)
- b) Solve  $(D^3 + 2D^2 + D)y = e^{3x} x^2 + \sin^2 x$  (6)
- c) Using fourth order Runge-Kutta method, solve numerically (8)
- $\frac{dy}{dx} = x^2 + y^2$  with the conditions  $x = 1, y = 1.5$  in the interval (1, 1.2) with  $h = 0.1$  correct to 4 decimals.
- 5 a) The density at any point of a cardioid  $r = a(1 + \cos\theta)$  varies as the (6)
- square of its distance from its axis of symmetry. Find its mass.
- b) An equation in the theory of stability of an aeroplane is (6)
- $\frac{dv}{dt} = g \cos\alpha - kv$   $v$  being velocity and  $g, k$  being constants. It is observed that at time  $t = 0$ , the velocity  $v = 0$ . Solve the equation.
- c) Evaluate  $\int_0^6 \frac{dx}{1+x^2}$  by using (i) Trapezoidal Rule, (ii) Simpson's  $(1/3)^{rd}$  (8)
- Rule and (iii) Simpson's  $(3/8)^{th}$  Rule. Also find the error.
- 6 a) Solve  $(2x+1)^2 \frac{d^2y}{dx^2} - 2(2x+1) \frac{dy}{dx} - 12y = 6x$  (6)
- b) For the curve  $x = a(2\cos t - \cos 2t), y = a(2\sin t - \sin 2t)$ , find (6)
- the length of the arc of the curve measured from  $t = 0$  to any point
- c) Find the volume cut off from the paraboloid  $x^2 + \frac{1}{4}y^2 + z = 1$  by the (8)
- plane  $z = 0$

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