**(Time: 3 hours)** **Max.Marks:80**

N.B (1) Question No.1 is compulsory
(2) Answer any three questions from Q.2 to Q.6
(3) Use of Statistical Tables permitted
(4) Figures to the right indicate full marks.

**1** a) Solve the equation $7\cosh x + 8\sinh x = 1$, for real values of x. **5**

b) Find $\alpha,\beta,\gamma$ when $A = \begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal. **5**

c) If $u = 3(ax+by+cz)^2 - \left(x^2+y^2+z^2\right)$ and $a^2+b^2+c^2 = 1$ show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2} = 0$ **5**

d) Find n[th] derivative of $y = \frac{x}{x^2+a^2}$ **5**

**2** a) *If* $\cos\alpha + 2\cos\beta + 3\cos\gamma = \sin\alpha + 2\sin\beta + 3\sin\gamma = 0$ *then prove that* $\sin 3\alpha + 8\sin 3\beta + 27\sin 3\gamma = 18\sin(\alpha+\beta+\gamma)$ **6**

b) If $v = \left(x^2 - y^2\right) f(xy)$, show that $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2} = \left(x^4 - y^4\right) f''(xy)$ **6**

c) If $y = e^{m\cos^{-1}x}$, then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$. Find $y_n(0)$ **8**

**3** a) Prove that $\sinh^{-1}(\tan x) = \log\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]$ **6**

b) Verify Euler's theorem for $u = \left(\frac{x^2+y^2}{x+y}\right)$ **6**

c) Examine the consistency of the system of equations $2x - y - z = 2,\ \ x + 2y + z = 2,\ \ 4x - 7y - 5z = 2$ and solve then if found consistent. **8**

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| 4 | a) | Find the real values of $\lambda$ for which the system has non-zero solutions. $x+2y+3z=\lambda x,\quad 3x+y+2z=\lambda y,\quad 2x+3y+z=\lambda z$ | 6 |
| | b) | Find the product of all the values of $\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)^{3/4}$ | 6 |
| | c) | If $u=\sin^{-1}\left[\left(x^2+y^2\right)^{1/5}\right]$ then show that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{2}{25}\tan u\left(2\tan^2 u-3\right)$ | 8 |
| 5 | a) | Using De Moivre's theorem, express $\frac{\sin 7\theta}{\sin\theta}$ in powers of $\sin\theta$ | 6 |
| | b) | If $xyz=8$ find the values of x,y,z for which $u=\frac{5xyz}{x+2y+4z}$ is maximum. | 6 |
| | c) | Considering only principle value, if $\left(1+i\tan\alpha\right)^{(1+i\tan\beta)}$ is real prove that its value is $\sec\alpha^{\sec^2\beta}$ | 8 |
| 6 | a) | Reduce to normal form and find its rank $A=\begin{bmatrix}3&2&5&7&12\\1&1&2&3&5\\3&3&6&9&15\end{bmatrix}$ | 6 |
| | b) | Find the extreme value of $u=x^3+xy^2+21x-2y^2-12x^2$ | 6 |
| | c) | Show that $\tan^{-1}\left(\frac{x+iy}{x-iy}\right)=\frac{\pi}{4}+\frac{i}{2}\log\left(\frac{x+y}{x-y}\right)$ | 8 |

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