University of Mumbai

Examinations Summer 2022

Time: 2 hour 30 minutes Max. Marks: 80

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Q1.	**Choose the correct options for following questions. All the Questions are compulsory and carry equal marks. 2 marks each.**
1.	The value of $\beta\left(\frac{5}{2},\frac{3}{2}\right)$ is equal to
Option A:	$\sqrt{\pi}$
Option B:	$\sqrt{2\pi}$
Option C:	π
Option D:	$\pi/16$
2.	The solution of differential equation $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=0$ is
Option A:	$(c_1+c_2x)c_3xe^{-x}$
Option B:	$(c_1+c_2x)e^{-2x}$
Option C:	$(c_1+c_2x)e^{2x}$
Option D:	$(c_1+c_2x)c_3xe^{x}$
3.	Particular Integral (P.I.) of differential equation $(D^3-3D^2)y=e^{4x}$ is
Option A:	$P.I.=\frac{1}{16}e^{-4x}$
Option B:	$P.I.=\frac{1}{64}e^{-4x}$
Option C:	$P.I.=\frac{1}{16}e^{4x}$
Option D:	$P.I.=\frac{1}{64}e^{4x}$
4.	Value of the integral $\int_0^\infty \int_0^\infty \int_0^\infty e^{-(x+y+z)}dx\,dy\,dz$ is
Option A:	∞
Option B:	0
Option C:	1
Option D:	-1
5.	The value of $\int_0^1 \int_0^{\pi/2} r\,sin\theta\,dr\,d\theta$ is
Option A:	½
Option B:	$\pi/2$
Option C:	$1/8$
Option D:	$\pi/8$
6.	Integrating factor of $(12y+4y^3+6x^2)dx+3(x+xy^2)dy=0$ is
Option A:	x^3
Option B:	x^2
Option C:	$logx$
Option D:	e^x
7.	The Value of $\int_0^\infty e^{-x^4}\,dx$ is given by
Option A:	$\Gamma\left(\frac{1}{4}\right)$
Option B:	$\frac{1}{4}\Gamma\left(\frac{3}{4}\right)$
Option C:	$\frac{1}{4}\Gamma\left(\frac{1}{4}\right)$
Option D:	$\Gamma\left(\frac{3}{4}\right)$

8.	The value of $I = \int_0^1 \int_0^1 \int_0^1 x\,y\,z\,dx\,dy\,dz$ is given by
Option A:	$-\frac{1}{8}$
Option B:	$\frac{1}{4}$
Option C:	$-\frac{1}{4}$
Option D:	$\frac{1}{8}$
9.	The value of I = $\int_0^1 \int_0^x (x^2+y^2)xdy\,dx$ is

Option A:	$\frac{3}{35}$
Option B:	$\frac{3}{15}$
Option C:	$\frac{1}{35}$
Option D:	$\frac{4}{15}$
10.	Length of the curve $y = logcosx$ from $x = 0\ to\ x = \frac{\pi}{3}$ is
Option A:	$log(1+\sqrt{2})$
Option B:	$log(2+\sqrt{3})$
Option C:	$log\,2$
Option D:	$log\,5$

Q2 **(20 Marks Each)**	**Solve any Four out of Six** **5 marks each**
A	Using Beta function, Prove that $\int_0^\infty \frac{1}{1+x^2}dx = \frac{\pi}{2}$
B	Using the method of variation of parameters, solve $\frac{d^2y}{dx^2} + a^2y = secax$
C	Show that the area between the parabolas $y^2 = 4ax \; and \; x^2 = 4by \; is \; \frac{16}{3}$ ab.
D	Solve y dx + x (1- $3x^2y^2$) $dy = 0$
E	Evaluate $\int_1^e \int_1^{logy} \int_1^{e^x} logz \; dzdxdy$
F	Find the perimeter of the cardioid r = a (1 -cosθ)

Q3 **(20 Marks Each)**	**Solve any Four out of Six** **5 marks each**
A	Solve $(D^3 - 2D^2 + D)y = x^2 + x$
B	Using beta and gamma function evaluate $\int_0^2 x^2 (2-x)^3 dx$
C	Change the order of integration for the integral and evaluate $\int_0^\infty \int_0^x xe^{\frac{-x^2}{y}} dxdy$
D	Solve $(x^2y - 2xy^2)$dx – $(x^3 - 3x^2y)dy = 0$
E	Express into polar form and evaluate the integral $I = \int_0^a \int_0^{\sqrt{a^2-x^2}} e^{-(x^2+y^2)} \, dx \, dy$
F	Evaluate the integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}} \, dx \, dy \, dz.$

Q4 **(20 Marks Each)**	**Solve any Four out of Six** **5 marks each**
A	Solve the following differential equation $y\frac{dy}{dx}+\frac{4x}{3}-\frac{y^2}{3x}=0$
B	Change to polar co-ordinates and evaluate $\int_0^1 \int_0^x x+y \; dydx$
C	Solve $(D^2+4)y = cos2x$
D	Evaluate using DUIS $\int_0^\infty \frac{log(1+ax^2)}{x^2}dx$
E	Evaluate the integral $\int \int \int \sqrt{x^2+y^2}\, dx\, dy\, dz$ over the region bounded by $x^2+y^2=z^2, z>0$ and $z=0, z=1.$
F	Evaluate $\int_0^{\frac{\pi}{6}} cos^3 3\theta \; sin^2 6\theta \; d\theta$