University of Mumbai

Examination First Half of 2022 (Summer-2022)

Program: _First Year (All Branches) Engineering-SEM-I

Program No - 1T01821

Applied Mathematics-I

Paper Code(58601)

Time: 2Hour 30 minutes **Marks: 80**

Q1. (20 Marks)	**Choose the correct option for following questions. All the Questions are compulsory and carry equal marks**
Q1.	If $y = \sin 2x \cos 3x$ then,
OptionA:	$y_n = \frac{1}{2}\left(5^n \sin\left(5x + \frac{n\pi}{2}\right) - \sin\left(x + \frac{n\pi}{2}\right)\right)$
OptionB:	$y_n = \frac{1}{2}\left(5^n \sin\left(5x + \frac{n\pi}{2}\right) + \sin\left(x + \frac{n\pi}{2}\right)\right)$
OptionC:	$y_n = \frac{1}{2}\left(5^n \sin\left(5x + \frac{n\pi}{2}\right) - \cos\left(x + \frac{n\pi}{2}\right)\right)$
OptionD:	$y_n = \frac{1}{2}\left(5^n \sin\left(5x + \frac{n\pi}{2}\right) + \cos\left(x + \frac{n\pi}{2}\right)\right)$
Q2.	Find the solution of following system of equations given by 2x-3y+7z=5, 3x+y-3z=13, 2x+19y-47z=32.
Option A:	No Solution Exist
Option B:	x=2, y=4, z=5
Option C:	x=2, y=-4, z=5
Option D:	x=-2, y=4, z=-5
Q3.	Find the rank of matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$.
Option A:	Rank (A)=1
Option B:	Rank (A)=2
Option C:	Rank (A)=3
Option D:	Rank (A)=0

Q4.	The system of equations 2x-2y+z=tx, 2x-3y+2z=ty, -x+2y=tz, will possess a solution for which values of constant t.
Option A:	t=1,3
Option B:	t=-1,3
Option C:	t=1,-3
Option D:	t=-1,-3
Q5.	If $\tanh x = 1/2$, then find the value of x and $\sinh 2x$
Option A:	$x = 1/2\log(3), \sinh 2x = 4/5$
Option B:	$x = -1/2\log(3), \sinh 2x = 4/5$
Option C:	$x = 1/2\log(3), \sinh 2x = 4/3$
Option D:	$x = -1/2\log(3), \sinh 2x = 4/3$
Q6.	For the unitary matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$, find A^{-1}
Option A:	$\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$
OptionB:	$\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
OptionC:	$\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$
OptionD:	$\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$
Q7.	If $u(x,y) = \tan^{-1}\left(\frac{x^2+y^2}{x-y}\right)$, then find the value of $I = x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.
Option A:	$I = \frac{1}{4}\sin u$
Option B:	$I = \frac{1}{4}\cos u$
Option C:	$I = \frac{1}{2}\sin 2u$
Option D:	$I = \frac{1}{4}\sin 2u$
Q8.	Simplify $\frac{(Cos3\theta + iSin3\theta)(Cos\theta - iSin\theta)}{(Cos5\theta - iSin5\theta)}$
Option A:	$(\cos 7\theta + i\sin 7\theta)$
Option B:	$(\cos 3\theta + i\sin 3\theta)$

Option C:	$(\cos 5\theta + i\sin 5\theta)$
Option D:	$(\cos\theta + i\sin\theta)$
Q9.	Find the maxima of $f = x^2 + y^2$, subjected to the condition $x + y = 2$.
Option A:	**2**
Option B:	**4**
Option C:	**5**
Option D:	**8**
Q10.	Find the value of $\log(\sqrt{3}\ -\ i)$
OptionA:	$log4 + i\frac{\pi}{6}$
OptionB:	$log2 + i\frac{\pi}{6}$
OptionC:	$log4 - i\frac{\pi}{6}$
OptionD:	$log2 - i\frac{\pi}{6}$

Q2. (20 Marks)	**Solve any Four out of Six (5 marks each)**
A	If $u = \log\tan(\frac{\pi}{4} + \frac{\theta}{2})$, Prove that i) $\text{Cosh}u = \sec\theta$, ii) $Sinhu = tan\theta$
B	$If\ u = \log(tanx + tany + tanz)\ prove\ that$ $Sin2x\frac{\partial u}{\partial x} + Sin2y\frac{\partial u}{\partial y} + Sin2z\frac{\partial u}{\partial z} = 2$
C	$Show\ that\ SinxSinhx = x^2 - 8\frac{x^6}{6!} + \cdots$
D	Prove that $log\left(1 + e^{i\theta}\right) = \log\left(2\,cos\frac{\theta}{2}\right) + i\frac{\theta}{2}$
E	Evaluate $\lim_{x\to 0}\frac{Sinx\sin^{-1}x - x^2}{x^6}$
F	Find the Rank of the following matrix by reducing to Normal Form $A = \begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$

Q3. (20 Marks)	**Solve any Four out of Six (5 marks each)**
A	*Show that* $\frac{Sin5\theta}{Sin\theta} = 16Cos^4\theta - 12Cos^2\theta + 1$
B	Test for consistency the following system & solve them if consistent $x_1 - 2x_2 + x_3 - x_4 = 2$ $x_1 + 2x_2 + 2x_4 = 1$ $4x_2 - x_3 + 3x_4 = -1$
C	Examine the function $u = x^3y^2(12 - 3x - 4y)$ For extreme values.
D	If $y^{1/m} + y^{-1/m} = x$ prove that $(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)\, y_n = 0$
E	Using Newton-Raphson method find the root of equation $2x^3 - 3x + 4 = 0$ lying between -2 and -1 correct to four places of decimals.
F	If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, then show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$

Q4. (20 Marks)	**Solve any Four out of Six (5 marks each)**
A	Show that minimum value of $u = xy + a^3(\frac{1}{x} + \frac{1}{y})$ is $3a^2$.
B	Find the n[th] derivative of $\frac{x}{1+3x+2x^2}$
C	*Solve* $x^5 = 1 + i$ *and find the continued product of the roots.*
D	Apply Gauss elimination method to solve the equations x+3y-2z=5, 2x+y-3z=1, 3x+2y-z=6.
E	If $u = tan^{-1}(\frac{x^2+y^2}{x-y})$ P.T $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -2sin^3u\, cosu$
F	For what value of λ the equations $x + 2y + z = 3, x + y + z = \lambda, 3x + y + 3z = \lambda^2$ have a solution and solve them completely in each case.