

University of Mumbai
Examinations Summer 2022
Program No: 1T01831

Examination: F.E. (Sem I) (ALL BRANCHES) (Rev 2019 'C'-Scheme)

Subject (Paper Code): 58651 // Engineering Mathematics - I

Time: 2 hour 30 minutes

Max. Marks: 80

Q I.	Choose the correct option for following questions. All the Questions are compulsory and carry equal marks.
1.	The value of $\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{10} + \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)^{10}$ is equal to
Option A:	$\frac{\pi}{2}$
Option B:	0
Option C:	$\frac{\pi}{3}$
Option D:	$\frac{\pi}{4}$
2.	What is the value of $\log(i)$
Option A:	$i \frac{\pi}{2}$
Option B:	0
Option C:	-2
Option D:	$-i \frac{\pi}{2}$
3.	If $u = \log(\tan x + \tan y)$ then the value of $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y}$ is
Option A:	2
Option B:	-1
Option C:	0
Option D:	-2
4.	All the stationary points of the function $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ are
Option A:	(6, 0), (4, 0), (5, 1), (5, -1)
Option B:	(6, 4), (4, 0), (5, 0), (5, 1)
Option C:	(6, 0), (0, 0), (5, 1), (5, -1)
Option D:	(0, 0), (4, 0), (5, 1), (5, -2)
5.	If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then rank of A is
Option A:	2
Option B:	3
Option C:	1
Option D:	0

6.	The modulus and principal value of the argument of $\frac{(1+i\sqrt{3})^{13}}{(\sqrt{3}-i)^{11}}$ is
Option A:	$\frac{1}{4}(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})$
Option B:	$4(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$
Option C:	$\frac{1}{4}(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$
Option D:	$4(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})$
7.	The real part of $\cos^{-1}(\frac{3i}{4})$ is
Option A:	π
Option B:	2π
Option C:	$-\pi$
Option D:	$\pi/2$
8.	If $u = \frac{\sqrt{xy}}{\sqrt{x}+\sqrt{y}}$ then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ is
Option A:	$\frac{u}{2}$
Option B:	$\frac{-u}{2}$
Option C:	$2u$
Option D:	$\sqrt{2}u$
9.	Stationary point is a point where $f(x, y)$ has
Option A:	$\frac{\partial f}{\partial x} = 0$
Option B:	$\frac{\partial f}{\partial y} = 0$
Option C:	$\frac{\partial f}{\partial x} = 0, \frac{\partial f}{\partial y} = 0$
Option D:	$\frac{\partial f}{\partial x} < 0, \frac{\partial f}{\partial x} > 0$
10.	For non-singular matrices P and Q, PAQ is in the normal form of a matrix A, then A^{-1} can be found by
Option A:	$A^{-1} = Q^{-1}P$
Option B:	$A^{-1} = P Q^{-1}$
Option C:	$A^{-1} = QP$
Option D:	$A^{-1} = Q P^{-1}$

Q II. (20 Marks)	Solve any Four out of Six.	5 marks each
A	Prove that: $\frac{\sin 6\theta}{\sin 2\theta} = 16\cos^2\theta - 16\cos^2\theta + 3$	
B	Considering only principal values separate into real and imaginary parts $i^{\log(1+i)}$.	
C	If $z = \tan^{-1}\left(\frac{y}{x}\right)$, find the value of $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$.	
D	Find the extreme value of the function $xy(3 - x - y)$.	
E	Express the matrix $\begin{bmatrix} 3i & -1+i & 3-2i \\ 1+i & -i & 1+2i \\ -3-2i & -1+2i & 0 \end{bmatrix}$ as a sum of Hermitian and skew Hermitian matrix.	
F	If $y = a \cos(\log x) + b \sin(\log x)$, then show that $x^2 y_{n+2} + (2n+1)x y_{n+1} + (n^2+1)y_n = 0$.	

Q III. (20 Marks)	Solve any Four out of Six.	5 marks each
A	Find all the values of $(1+i)^{\frac{1}{3}}$ and show that their continued product is $(1+i)$.	
B	Separate into real and imaginary parts $\tan^{-1}(\alpha + i\beta)$	
C	If $u = \sin^{-1} \frac{x+y}{\sqrt{x}+\sqrt{y}}$, prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4\cos^3 u}$.	
D	Divide 24 into 3 parts such that the continued product of the first, square of second and cube of the third is maximum using Lagrange's method.	
E	Find a, b, c if A is orthogonal matrix where $A = \frac{1}{3} \begin{bmatrix} a & b & c \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$. Hence find inverse of A.	
F	Investigate for what values of λ and μ the system of equations $x + y + z = 6$; $x + 2y + 3z = 10$; $x + 2y + \lambda z = \mu$ has (i) no solution (ii) a unique solution (iii) an infinite no. of solutions.	

Q IV. (20 Marks)	Solve any Four out of Six.	5 marks each
A	Prove that $(1 + i\sqrt{3})^n + (1 - i\sqrt{3})^n = 2^{n+1} \cos \frac{n\pi}{3}$.	
B	Prove that $\sinh^{-1}(\tan \theta) = \log \left[\tan \left(\frac{\pi}{4} + \frac{\theta}{2} \right) \right]$	
C	If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, then show that $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0$	
D	Find n^{th} derivatives of $\frac{x}{(x-1)(x-2)(x-3)}$.	

E	Find non-singular matrices P and Q such that $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ is reduced to normal form. Also find its rank.
F	Using De Moivre's theorem prove that $\cos^6 \theta - \sin^6 \theta = \frac{1}{16} (\cos 6\theta + 15 \cos 2\theta).$