(Time: 3 hours)   Max.Marks:80

N.B   (1) Question No.1 is compulsory
(2) Answer any three questions from Q.2 to Q.6
(3) Use of Statistical Tables permitted
(4) Figures to the right indicate full marks.

1
a) Prove that $\sec h^{-1}(\sin \theta) = \log\left(\cot \dfrac{\theta}{2}\right)$ 5

b) If $z = x^y + y^x$ then prove that $\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{\partial^2 z}{\partial y \partial x}$ 5

c) If α, β are the roots of the quadratic equation $x^2 - 2\sqrt{3}\, x + 4 = 0$,   5
find the value of $\alpha^3 + \beta^3$

d) Test the consistency and if possible solve   5
$2x - 3y + 7z = 5$, $3x + y - 3z = 13$, $2x + 19y - 47z = 32$

2
a) Is $A = \begin{bmatrix} \dfrac{2+i}{3} & \dfrac{2i}{3} \\ \dfrac{2i}{3} & \dfrac{2-i}{3} \end{bmatrix}$ a unitary matrix?   6

b) Find the nth derivative of $y = \dfrac{4x}{(x-1)^2(x+1)}$ 6

c) If $u = \dfrac{x^4 + y^4}{x^2 y^2}$ then find the value of   8

$x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2}$ at $x = 1$ and $y = 2$

3 a) Prove that $\log(1 + \cos 2\theta + i\sin 2\theta) = \log(2\cos\theta) + i\theta$ 6

b) Solve $x^7 + x^4 + i(x^3 + 1) = 0$ using De Moivre's theorem   6

c) Discuss for all values of k for which the system of equations   8
has a non-trivial solution
$2x + 3ky + (3k + 4)z = 0$,
$x + (k + 4)y + (4k + 2)z = 0$, $x + 2(k + 1)y + (3k + 4)z = 0$

4 a) If $u = \log(r)$ and $r = x^3 + y^3 - x^2 y - xy^2$ then show that   6

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$$

b) Find two non-singular matrices P and Q such that PAQ is in   6

the normal form   where $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$

c) Prove that $\tan^{-1} \left(e^{i\theta} \right) = \frac{n\pi}{2} + \frac{\pi}{4} - \frac{i}{2} \log \tan \left(\frac{\pi}{4} - \frac{\theta}{2} \right)$ 8

5 a) Considering principal value, express in the form $a + ib$ the   6
quantity $\left(\sqrt{i} \right)^{\sqrt{i}}$

b) Prove that $\tan 5\theta = \dfrac{5 \tan \theta - 10 \tan^3 \theta + \tan^5 \theta}{1 - 10 \tan^2 \theta + 5 \tan^4 \theta}$ 6

c) If $y = e^{a \sin^{-1} x}$, then Prove that   8
$\left(1 - x^2 \right) y_{n+2} - \left(2n + 1 \right) xy_{n+1} - \left(n^2 + a^2 \right) y_n = 0$ Also find $y_n(0)$

6 a) If $u = \dfrac{1}{r}$, $r = \sqrt{x^2 + y^2 + z^2}$ then prove that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = 0$ 6

b) If $\dfrac{3}{x} + \dfrac{4}{y} + \dfrac{5}{z} = 6$ find the values of x, y, z such that $x + y + z$ is   6
minimum

c) Prove that every Skew-Hermitian matrix can be expressed in   8
the form B+iC, where B is real Skew-Symmetric and C is real
Symmetric matrix and express the matrix

$A = \begin{bmatrix} 2i & 2+i & 1-i \\ -2+i & -i & 3i \\ -1-i & 3i & 0 \end{bmatrix}$ as B+iC where B is real Skew-

symmetric matrix and C is real Symmetric matrix