(3 Hours) [Total marks : 80

Note : -
1) Question number **1** is **compulsory.**
2) Attempt any **three** questions from the remaining **five** questions.
3) **Figures** to the **right** indicate **full** marks.

Q.1 a) If $u = \log\left(\frac{x}{y}\right) + \log\left(\frac{y}{x}\right)$, find $\frac{\partial u/\partial x}{\partial u/\partial y}$ 03

b) Find the value of $\tanh(\log x)$ if $x = \sqrt{3}$ 03

c) Evaluate $\lim_{x \to 3}\left[\frac{1}{x-3} - \frac{1}{\log(x-2)}\right]$ 03

d) If $u = r^2\cos 2\theta$, $v = r^2\sin 2\theta$, find $\frac{\partial(u,v)}{\partial(r,\theta)}$ 03

e) Express the matrix $A = \begin{pmatrix} 2+3i & 2 & 3i \\ -2i & 0 & 1+2i \\ 4 & 2+5i & -i \end{pmatrix}$ as the sum of a 04

Hermitian and a Skew-Hermitian matrix.

f) Expand $\tan^{-1}x$ in powers of $\left(x - \frac{\pi}{4}\right)$ 04

Q.2 a) Expand $\sin^7\theta$ in a series of sines of multiples of θ 06

b) If $y = \sin^2 x\cos^3 x$, find y_n 06

c) Find the stationary values of $x^3 + y^3 - 3axy$, $a > 0$ 08

Q. 3 a) Compute the real root of $x\log_{10}x - 1.2 = 0$ correct to three places of decimals using Newton-Raphson method. 06

b) Show that the system of equations
$2x - 2y + z = \lambda x$, $2x - 3y + 2z = \lambda y$, $-x + 2y = \lambda z$
can posses a non-trivial solution only if $\lambda = 1, \lambda = -3$. Obtain the general solution in each case. 06

c) If $\tan(\alpha + i\beta) = \cos\theta + i\sin\theta$, prove that
$\alpha = \frac{n\pi}{2} + \frac{\pi}{4}$ and $\beta = \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$ 08

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Q. 4   a)   Using the encoding matrix as $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, encode and decode the message   06

           MOVE

      b)   If $u = f(e^{x-y}, e^{y-z}, e^{z-x})$ then prove that   06

           $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$

      c)   If $y = a \cos(\log x) + b \sin(\log x)$, then show that   08

           $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2 + 1)y_n = 0$

Q. 5   a)   If $1, \alpha, \alpha^2, \alpha^3, \alpha^4$, are the roots of $x^5 - 1 = 0$, find them and show   06

           that $(1-\alpha)(1-\alpha^2)(1-\alpha^3)(1-\alpha^4) = 5$

      b)   If $\theta = t^n e^{-r^2/(4t)}$,   06

           Find n which will make $\dfrac{\partial \theta}{\partial t} = \dfrac{1}{r^2} \dfrac{\partial}{\partial r}\left(r^2 \dfrac{\partial \theta}{\partial r}\right)$

      c)   Find the root (correct to three places of decimals) of $x^3 - 4x - 9 = 0$ 08

           lying between 2 and 3 by using Regula-Falsi method.

Q. 6   a)   Find non-singular matrices P and Q such that   06

           $A = \begin{pmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{pmatrix}$ is reduced to normal form. Also find its rank.

      b)   Find the principle value of $(1+i)^{1-i}$ 06

      c)   Solve the following equations by Gauss-Seidel method   08

           $27x + 6y - z = 85$

           $6x + 15y + 2z = 72$

           $x + y + 54z = 110$

           (Take three iterations)

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