(3 Hours) **[Total Marks : 80**

Note:

1) Question No.1 is compulsory
2) Attempt any three out of remaining five questions
3) Figures to the right indicate full marks

Q1.

a) If $\sin(\theta + i\varphi) = tan\alpha + isec\alpha$, then show that $\cos 2\theta \,.\, cosh2\varphi = 3$ **[5]**

b) If $u = \log(tanx + tany)$, then show that $sin2x\frac{\partial u}{\partial x} + sin2y\frac{\partial u}{\partial y} = 2$ **[5]**

c) Express the matrix A = $\begin{bmatrix} 0 & 5 & -3 \\ 1 & 1 & 1 \\ 4 & 5 & 9 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix. **[5]**

d) Expand $\sqrt{1 + sinx}$ in ascending powers of x upto x^4 term. **[5]**

Q2.

a) Find non-singular matrices P and Q such that PAQ is in normal form where, **[6]**

A= $\begin{bmatrix} 4 & 3 & 1 & 6 \\ 2 & 4 & 2 & 2 \\ 12 & 14 & 5 & 16 \end{bmatrix}$. Also find the rank of A.

b) If $z = f(x, y)$ and $x = u\, coshv,\ y = u\, sinhv$; prove that **[6]**

$$\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial u}\right)^2 - \frac{1}{u^2}\left(\frac{\partial z}{\partial v}\right)^2$$

c) Prove that $Log\left[\frac{(a-b)+i(a+b)}{(a+b)+i(a-b)}\right] = i(2n\pi + tan^{-1}\frac{2ab}{a^2-b^2})$. Hence evaluate $Log\left(\frac{1+5i}{5+i}\right)$ **[6]**

Q3.

a) If $\alpha \ and \ \beta$ are the roots of the equation $z^2 sin^2\theta - z \ sin \ 2\theta + 1 = 0$, then prove that
$\alpha^n + \beta^n = 2\cos n\theta \ cosec^n\theta$ and $\alpha^n\beta^n = cosec^{2n}\theta$ **[6]**

b) Solve the following equations by Gauss–Seidal Method ; **[6]**
$15x + 2y + z = 18$, $2x + 20y - 3z = 19$, $3x - 6y + 25z = 22$,
Take three iterations.

c) Prove that if z is a homogeneous function of two variables x and y of degree n, then $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z$. Hence find the value of $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 Z}{\partial y^2}$ at $x = 1, y = 1$ when z $= x^6 \ tan^{-1}\left(\frac{x^2+y^2}{x^2+xy}\right) + \frac{x^4+y^4}{x^2y^2}$ **[8]**

Q4.

a) If tan $(\alpha + i\beta) = \cos\theta + i\sin\theta$ then prove that $\alpha = \frac{n\pi}{2} + \frac{\pi}{4}, \beta = \frac{1}{2}$ log $\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$ **[6]**

b) Expand $x^5 + x^3 - x^2 + x - 1$ in powers of $(x-1)$ and hence find the value of **[6]**

1) $f\left(\frac{9}{10}\right)$
2) $f(1.01)$

c) For what values of λ and μ , the equations, **[8]**
$x + y + z = 6$; $x + 2y + 3z = 10$; $x + 2y + \lambda z = \mu$

1) have a unique solution
2) have infinite solution

Find the solution in each case for a possible value of μ and λ.

Q5.

a) Find the nth derivative of $y = \dfrac{1}{x^2 + a^2}$ **[6]**

b) Discuss the maxima and minima of $x^3 + xy^2 - 12x^2 - 2y^2 + 21x + 16$ **[6]**

c) Prove that if A and B are two unitary matrices then AB is also unitary. Verify the result when

$A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ and $B = \begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$ **[8]**

Q6.

a) If $x = \cosh\left(\frac{1}{m}\log y\right)$, prove that **[6]**

$(x^2 - 1)y_{n+2} + (2n+1)x\, y_{n+1} + (n^2 - m^2)\, y_n = 0$

b) Find a root of the equation $xe^x = \cos x$ using the Regular Falsi Method correct to three decimal places. **[6]**

c) 1) Expand $sin^4\theta\, cos^2\theta$ in a series of multiples of θ. **[4]**

2) If one root of $x^4 - 6x^3 + 18x^2 - 24x + 16 = 0$ is (1+i); find the other roots. **[4]**
