

(Time: 3 hours)

Total Marks: 80

- N.B. (1) Question no. 1 is Compulsory
(2) Solve any three from the remaining.

Q.1) a) Prove that $(1 + i\sqrt{3})^8 + (1 - i\sqrt{3})^8 = -2^8$ (3)

b) If $A = \frac{1}{3} \begin{pmatrix} 1 & 2 & a \\ 2 & 1 & b \\ 2 & -2 & c \end{pmatrix}$ is orthogonal find a,b,c. (3)

c) $z^3 + xy - y^2z = 6$ find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ where z is an implicit function of x and y. (3)

d) If $u = e^x \cos y, v = e^x \sin y$ find $\frac{\partial(u,v)}{\partial(x,y)}$. (3)

e) Find the n^{th} derivative of $y = \frac{x^2+4x+1}{x^3+2x^2-x-2}$ (4)

f) Find a,b if $\lim_{x \rightarrow 0} \frac{a \sinh x + b \cosh x}{x^3} = \frac{5}{3}$ by L'Hospital's Rule. (4)

Q.2) a) Find the roots common to $x^4 + 1 = 0$ and $x^6 - i = 0$ (6)

b) If $y = \sin^{-1}x$ Prove That $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$ also find $y_9(0)$ (6)

c) Discuss the maxima and minima of $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ (8)
Hence find maximum and minimum value of $f(x, y)$.

Q.3) a) Find the values of k for which the equations $x + y + z = 1, x + 2y + 3z = k, x + 5y + 9z = k^2$ have a solution, solve them for these values of k. (6)

b) If $x = \sqrt{vw}, y = \sqrt{wu}, z = \sqrt{uv}$, Prove that $x \frac{\partial \phi}{\partial x} + y \frac{\partial \phi}{\partial y} + z \frac{\partial \phi}{\partial z} = u \frac{\partial \phi}{\partial u} + v \frac{\partial \phi}{\partial v} + w \frac{\partial \phi}{\partial w}$ where ϕ is the function of x,y,z. (6)

c) If $\tan(\alpha + i\beta) = \cos\theta + i\sin\theta$ Prove that $\alpha = \left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$ & $\beta = \frac{1}{2} \log \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$. (8)

Q.4) a) If $z = e^{x/y} + \log(x^3 + y^3 - x^2y - xy^2)$, Find the value of $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} + x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2}$. (6)

b) Using encoding matrix $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ encode and decode the message NOW*STUDY (6)

c) Solve the following equations by Gauss Jacobi's Iteration method $15x + 2y + z = 18, 2x + 20y - 3z = 19, 3x - 6y + 25z = 22$ (8)

Q.5) a) Prove that the general value of $(1 + itana)^{-i}$ is $e^{2m\pi+\alpha}[\cos(\log\cos\alpha) + i\sin(\log\cos\alpha)]$ (6)

b) State and Prove Eulers Theorem for function of Three Variables. (6)

c) Expand $x^5 - x^4 + x^3 - x^2 + x - 1$ in powers of $(x - 1)$ and hence find $f\left(\frac{11}{10}\right), f(0.99)$. (8)

Q.6) a) Prove that:

$$\sinh^7 x = \frac{1}{64}(\sinh 7x - 7\sinh 5x + 21\sinh 3x - 35\sinh x) \quad (6)$$

b) Find nonsingular matrices P and Q such that PAQ is in Normal form.

also find Rank of A, where $A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 2 & 2 \\ 7 & 4 & 10 \\ 1 & 0 & 6 \end{bmatrix}$. (6)

c) Using Newton Raphson Method find an iterative formula for $\sqrt[N]{N}$ where N is positive number, Hence find $\sqrt[5]{35}$. (8)
