[Time: 3 Hours] **[Marks:80]**

Please check whether you have got the right question paper.

N.B: 1. Question 1 is compulsory.
2. Attempt any three questions from Q.2 to Q.6.

Q1 (a) Prove $cosh^5 x = \cosh 5x + 5\cosh 3x + 10\cosh x$ **(3)**

(b) If u= log(tanx+ tany) **(3)**

Prove $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} = 2$

(c) **(3)**

If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$ Show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$

(d) Express the following matrix as sum of symmetric and skew symmetric matrix. **(3)**

$$A = \begin{pmatrix} 2 & 2+i & 3 \\ -2+i & 0 & 4i \\ -i & 3-i & 1-i \end{pmatrix}$$

(e) Show that $\log(1+\sin x) = x - \frac{x^2}{2} + \frac{x^3}{6}$ **(4)**

(f) If $y = \frac{x^2}{(x-1)(x-2)}$ Find y_n **(4)**

Q2 (a) Solve the Equation $x^4 - x^3 + x^2 - x + 1 = 0$ **(6)**

(b) Reduce the following Matrix to the Normal form and hence find the rank of the matrix **(6)**

$$A = \begin{pmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{pmatrix}$$

(c) If $u = \frac{x^2y^2z^2}{x^2+y^2+z^2} + \cos^{-1}\left(\frac{xy+yz}{\sqrt{x^2+y^2+z^2}}\right)$ **(8)**

Find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$

Q3 (a) (a) Investigate for what values of λ and μ the system of equations **(6)**
x+2y+3z= 4, x+3y+4z = 5, x+3y+λz =μ.
have 1) unique solution, 2) Infinite solutions, 3) No solution

(b) Find the Extreme values of f(x,y)= xy+ $a^3(\frac{1}{x}+\frac{1}{y})$ **(6)**

(c) Separate into real and imaginary parts of $\tan^{-1}(e^{i\theta})$ **(8)**

Q4 (a) If $u^2+xv^2=x+y$, $v^2+yu^2=x-y$ Find $\frac{\partial u}{\partial x},\frac{\partial v}{\partial y}$ **(6)**

(b) If log cos(x+iy)= a+ib Prove $2e^{2a}$= cosh2y +cos 2x **(6)**

(c) **(8)**
Solve the following Equations by Gauss Seidel method Upto four iterations.
4x-2y-z= 40 , x-6y+2z= -28 , x-2y+12z= -86

Q5 (a) Using De Moivre's theorem Prove **(6)**
$cos^7\theta = \frac{1}{2^6}(\cos 7\theta + 7\cos 5\theta + 21\cos 3\theta + 35\cos\theta)$

(b) Evaluate $\lim_{x\to 0}\left(\frac{1}{x^2}-cot^2x\right)$ **(6)**

(c) If y = $\sin(m\sin^{-1}x)$ Prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}+(m^2-n^2)y_n=0$ **(8)**
And hence find$y_3(0)$.

Q6 (a) (a) Show the following vectors are linearly dependent and find the relation between them. **(6)**
$[2,-1,3,2]$,$[1,3,4,2]$,$[3,-5,2,2]$.

(b) If z=f(x,y) where x = u cosh v ,y = usinh v Prove **(6)**
$$\left(\frac{\partial z}{\partial x}\right)^2-\left(\frac{\partial z}{\partial y}\right)^2=\left(\frac{\partial z}{\partial u}\right)^2-\frac{1}{u^2}\left(\frac{\partial z}{\partial v}\right)^2$$

(c) Fit the curve of the form y= ab^x to the following data. **(8)**

x	2	3	4	5	6
y	144	172.8	207.4	248.8	298.5

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