

(3 hours)

Total Marks: 80

Note:

1. Question no.1 is compulsory
2. Answer any three from remaining

1. a. Show that $\operatorname{sech}^{-1}(\sin \theta) = \log \cot \left(\frac{\theta}{2} \right)$ (3)

b. Show that the matrix $A = \frac{1}{2} \begin{pmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{pmatrix}$ is unitary (3)

c. Evaluate $\lim_{x \rightarrow 0} \sin x \log x$ (3)

d. Find the nth derivative of $y = e^{ax} \cos^2 x \sin x$ (3)

e. If $x = r \cos \theta$ and $y = r \sin \theta$ prove that $JJ' = 1$ (4)

f. Using coding matrix $A = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$ encode the message (4)

THE CROW FLIES AT MIDNIGHT

2. a. Find all values of $(1 + i)^{\frac{1}{3}}$ and show that their continued product is $(1 + i)$ (6)

b. Find the non singular matrices P & Q such that PAQ is in normal (6)

form where $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{pmatrix}$

c. Find max. and minimum values of $x^3 + 3x^2y - 15x^2 - 15y^2 + 72x$ (8)

3. a. If $u = e^{xyz} f\left(\frac{xy}{z}\right)$ prove that $x \frac{\partial u}{\partial x} + z \frac{\partial u}{\partial z} = 2xyz u$ (6)

and $y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 2xyz u$ and hence show that

$$x \frac{\partial^2 u}{\partial z \partial x} = y \frac{\partial^2 u}{\partial z \partial y}$$

b. By using Regular falsi method solve $2x - 3\sin x - 5 = 0$ (6)

correct to three decimal places

c. If $y = \sin [\log(x^2 + 2x + 1)]$ then prove that (8)

$$(x + 1)^2 y_{n+2} + (2n + 1)(x + 1)y_{n+1} + (n^2 + 4)y_n = 0$$

4. a. State and prove Eulers Theorem for three variables. (6)

b. By using De Moivres Theorem obtain $\tan 5\theta$ in terms of (6)

$$\tan \theta \text{ and show that } 1 - 10 \tan^2 \left(\frac{\pi}{10} \right) + 5 \tan^4 \left(\frac{\pi}{10} \right) = 0$$

c. Investigate for what values of λ and μ the equations (8)

$$2x + 3y + 5z = 9$$

$$7x + 3y - 2z = 8$$

$$2x + 3y + \lambda z = \mu \text{ have}$$

- (i) No solution
- (ii) Unique solution
- (iii) An infinite number of solution

5. a. Find nth derivative of $\frac{1}{x^2 + a^2}$ (6)

b. If $z = f(x, y)$ where $x = e^u + e^{-v}$, $y = e^{-u} - e^v$ then (6)

$$\text{prove that } \frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}$$

c. Solve by using Gauss Jacobi Iteration method (8)

$$2x + 12y + z - 4w = 13$$

$$13x + 5y - 3z + w = 18$$

$$2x + y - 3z + 9w = 31$$

$$3x - 4y + 10z + w = 29$$

6. a. If $y = \log \left[\tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right]$ Prove that (6)

(i) $\tan h \frac{y}{2} = \tan \frac{x}{2}$

(ii) $\coth y \cos x = 1$

b. If $u = \sin^{-1} \left[\frac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}} \right]^{1/2}$ prove that (6)

$$x^2 \frac{\partial^2 u}{\partial^2 x} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial^2 y} = \frac{\tan u}{144} [\tan^2 u + 13]$$

c.(i) Expand $2x^3 + 7x^2 + x - 6$ in powers of (4)

$(x - 2)$ by using Taylors theorem.

(ii) Expand $\sec x$ by Maclaurins theorem considering upto x^4 term (4)
