

(3 hours)

Total marks: 80

N.B.: (1) Question No. 1 is compulsory

(2) Attempt any Three from remaining

- Q1 a) If $\log \tan x = y$ then prove that $\sinh ny = \frac{1}{2} [\tan^n x - \cot^n x]$ [3]
- b) If $u = x^2y + e^{xy^2}$ Find $\frac{\partial^2 u}{\partial x \partial y}$ [3]
- c) If $x = u - uv$, $y = uv - uvw$, $z = uvw$ find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ [3]
- d) Using Maclaurin's series, Prove $e^{ex} = e + ex + ex^2 + \dots$ [3]
- e) Show that $A = \begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$ is unitary [4]
if $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$
- f) Find n^{th} derivative of $\frac{x}{(x-1)(x-2)(x-3)}$ [4]

- Q2 a) Solve $x^5 = 1 + i$ and find the continued product of the roots. [6]
- b) Reduce the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ to the normal form [6]
and find its Rank
- c) State and Prove Euler's theorem for two variables hence [8]
find value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ where $u = \frac{\sqrt{xy}}{\sqrt{x} + \sqrt{y}}$

- Q3 a) Test the consistency of [6]
 $2x - y - z = 2$, $x + 2y + z = 2$, $4x - 7y - 5z = 2$
And Solve if consistent.
- b) Examine the function for its extreme values [6]
 $f(x, y) = y^2 + 4xy + 3x^2 + x^3$
- c) If $\sin(\theta + i\phi) = e^{i\alpha}$ then Prove $\cos^4 \theta = \sin^2 \alpha = \sinh^4 \phi$ [8]

- Q4 a) If $x = u \cos v$, $y = u \sin v$ then [6]
Prove $\frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(u,v)} = 1$
- b) If $\log(x + iy) = e^p (\cos q + i \sin q)$ then [6]
prove that $y = x \tan(\tan q \cdot \log \sqrt{x^2 + y^2})$
- c) Solve by Gauss Elimination method [8]
 $2x + 3y + 4z = 11$, $x + 5y + 7z = 1$, $3x + 11y + 13z = 25$

Q5 a) Prove $\cos^6 \theta + \sin^6 \theta = \frac{1}{8} [3 \cos 4\theta + 5]$ [6]

b) Evaluate $\lim_{x \rightarrow 0} \left[\frac{1}{x^2} - \cot^2 x \right]$ [6]

c) If $y = \cos(m \sin^{-1} x)$ then [8]
 prove that $(1 - x^2)y_{n+2} - (2n + 1)x y_{n+1} + (m^2 - n^2)y_n = 0$

Q6 a) Check if the following vectors [6]

$$X_1 = [1, 0, 2, 1], X_2 = [3, 1, 2, 1], X_3 = [4, 6, 2, -4],$$

$X_4 = [-6, 0, -3, -4]$ are linear dependent hence find the relation between them if any.

b) If $f(xy^2, z - 2x) = 0$ then [6]
 prove that $2x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 4x$

c) Fit a second degree parabola $y = ax^2 + bx + c$ to the following data [8]

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9