(3 Hours) **Total Marks: 80**

N.B. : **(1)** Question No. 1 is compulsory.
(2) Attempt any three questions out of remaining five questions

Q.1. (a) Discuss the values of a and b, for which the following system of equations has (a)no solution (b) a unique solution and (c) infinite number of solutions : **(5)**

$$x + 2y + 3z = 0$$
$$x + 3y + 5z = 9$$
$$2x + 5y + az = b$$

(b) The joint probability mass function of two discrete variables X and Y are given by **(5)**
P(X=x,Y=y)=c(2x+y) where X and Y can assume all integer in the range
$0 \le x \le 2, \quad 0 \le y \le 3$
(i) Find the value of c
(ii) Marginal density function of X

(c) Obtain the Graph of y=log(2x) **(5)**

(d) Find the stationary points of the function **(5)**

$$Z = 2x_1 + x_3 + 3x_2x_3 - x_1^2 - 3x_2^2 - 3x_3^2 + 17$$

Q.2. (a) Find the dimension and basis for the four fundamental subspaces for **(10)**
$A=\begin{bmatrix}3 & 1\\ 5 & 2\end{bmatrix}$

(b) A random sample of 395 people were surveyed and each person was asked to report the highest education level they obtained. The data that resulted from the survey is summarized in the following table: **(10)**

	High School	**Bachelors**	**Masters**	**Ph.d.**	**Total**
Female	60	54	46	41	201
Male	40	44	53	57	194
Total	100	98	99	98	395

Are gender and education level dependent at 5% level of significance?

(Given $\chi^2(3,0.05) = 7.815$)

Q.3. (a) The following table gives the random sample of marks obtained by students in two schools, A and B **(10)**

School A	63	72	80	60	85	83	70	72	81
School B	86	93	64	82	81	75	86	63	63

Is the variance of Marks of the students in School A is less than that of those in School B? Test at 5% level of significance.
(Given $F_{((8,8),0.95)=0.291}$)

(b) Explain types of data. Compare nominal and ordinal data **(10)**

Q.4. (a) What is Ogive. Construct less than Ogive and greater than Ogive using the give data **(10)**

Class	Frequency
100-120	2
120-130	5
130-140	9
140-150	14
150-160	7
160-170	3

(b) Discuss the need for exploratory data analysis and explain EDA techniques **(10)**

Q.5. (a) Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ starting from the point X1=(0,0) using steepest descent method. **(10)**

(b) Minimize the function $f(x_1, x_2) = 4x_1 + 8x_2 - x_1^2 - x_2^2$ subject to $x_1 + x_2 = 4, \quad x_1, x_2 \geq 0$ **(10)**

Q.6. Write short notes on (any four) **(20)**

(a) Four Fundamental Subspaces **(5)**

(b) Non linear dimensionality reduction-Multidimensional Scaling **(5)**

(c) Principal Component Analysis (PCA) algorithm. **(5)**

(d) 5 Number Summary plot **(5)**

(e) Gradient based optimization Techniques **(5)**

(f) Machine learning algorithms **(5)**