

Duration: 3 hours

Max. Marks: 80

N.B. (1) Question No. 1 is **COMPULSORY**.(2) Answer **ANY THREE** questions from Q.2 to Q.6.

(3) Use of Statistical Tables permitted.

(4) Figures to right indicate full marks.

**Q.1** a. Evaluate the complex line integral  $\int_0^{1+i} (x^2 - iy) dz$  along the straight line from  $z=0$  to  $z=1+i$  5

b. Check whether the set of vectors  $(1, 4, 8)$ ,  $(2, 5, 6)$ ,  $(3, 1, -4)$  are basis for  $\mathbb{R}^3$  5

c. A discrete random variable 'X' has following distribution 5

|      |   |     |    |    |    |    |   |
|------|---|-----|----|----|----|----|---|
| X    | 0 | 1   | 2  | 3  | 4  | 5  | 6 |
| P(X) | k | 15k | 8k | 7k | 5k | 3k | k |

Find the value of k and mean of X

d. Find rank correlation coefficient of the following data 5

|   |    |    |    |    |    |    |    |    |    |
|---|----|----|----|----|----|----|----|----|----|
| X | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| Y | 13 | 14 | 12 | 18 | 23 | 17 | 15 | 16 | 21 |

**Q.2** a. Find the extremal of the function  $\int_0^{\pi/2} (y'^2 - y^2 + 2xy) dx$  with  $y(0)=0$  and  $y(\pi/2) = 0$  6

b. Fit a straight line of the form  $y=a+bx$  to the following data 6

|   |   |    |    |    |    |    |
|---|---|----|----|----|----|----|
| X | 1 | 3  | 5  | 7  | 8  | 10 |
| Y | 8 | 12 | 15 | 17 | 18 | 20 |

c. Obtain Laurent series expansion of function 8

$$f(z) = \frac{2}{z^2 - 3z + 2} \text{ in the region i. } |z| < 1 \text{ ii. } |z| > 2 \text{ iii. } 1 < |z| < 2$$

**Q.3** a. A continuous random variable has probability density function as  $f(x) = kx^2(1-x)$   $0 \leq x \leq 1$ , find k, mean and variance 6

b. Find the Extremal of  $\int_0^1 y y' + (y'')^2 dx$ , 6

c. Reduce the quadratic form to canonical form, find its rank and signature 8

$$x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 - 2x_1x_3 + 2x_3x_2$$

- Q.4**
- Using Gram-Schmidt process, construct, an orthonormal basis of  $(1, 0, 1, 1), (-1, 0, -1, 1)$  and  $(0, -1, 1, 1)$  have Euclidian inner product **6**
  - An insurance company found that only 0.01% of the population is involved in a certain type of accident each year. If its 1000 policy holders were randomly selected from the population, what is the probability that not more than two of its clients are involved in such accident next year. **6**
  - By Rayleigh -Ritz method, Solve the boundary value problem  $y''+y+x=0$   $0 < x < 1$   $y(0)=y(1)=0$  **8**

- Q.5**
- Ten students got the following percentage of marks in mathematics and statistics **6**

|       |    |    |    |    |    |    |    |    |    |    |
|-------|----|----|----|----|----|----|----|----|----|----|
| Maths | 78 | 36 | 98 | 25 | 75 | 82 | 90 | 62 | 65 | 39 |
| Stats | 84 | 51 | 91 | 60 | 68 | 62 | 86 | 58 | 53 | 47 |

Calculate the coefficient of correlation

- In a normal distribution 17% of the items are below 30 and 17% of the items are above 60. Find the mean and standard deviation. **6**
  - Find Singular value decomposition of  $\begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$  **8**
- Q.6**
- Verify Cauchy -Schwartz inequality for  $u=(1, 3, 2, 4)$  and  $v=(3, -2, 1, 6)$  **6**
  - Using Cauchy residue theorem evaluate  $\int_C \frac{z-1}{(z+1)^2(z-2)} dz$  where  $C$  is  $|z-i|=2$  **6**
  - Obtain the equations of the lines of regression for the following data. Also obtain the estimate of  $X$  for  $Y=70$ . **8**

|   |    |    |    |    |    |    |    |    |
|---|----|----|----|----|----|----|----|----|
| X | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 |
| Y | 67 | 68 | 65 | 68 | 72 | 72 | 69 | 71 |