Duration: 3 hours Max. Marks: 80

N.B. (1) Question No. 1 is **COMPULSORY**.

- (2) Answer **ANY THREE** questions from Q.2 to Q.6.
- (3) Use of Statistical Tables permitted.
- (4) Figures to right indicate full marks.

Q 1
a. Evaluate the complex line Integral
$$\int_{0}^{1+i} (x-y+ix^{2})dz$$
 along the straight line from z=0 to z=1+i

- b. Find a vector orthogonal to u=(-6, 4, 2) v=(3, 1, 5)
- The equations of lines of regressions are 2x+3y+8=0 and x+2y-5=0,
 c. find means of x and y and coefficient of line of regression between x and y
- Let W be the set of 2 X 2 matrices of the form $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ where a and b d. are real numbers, Show that W is a subspace of space V of all 2X 2 matrices.
- Q 2 Find the Spearman's rank coefficient correlation of the following data 6

| | | | | | Qn' | | | 425 | | | |
|----|--------------|----|----|----|-----|----|----|-----|----|----|----|
| a. | \mathbf{X} | 32 | 55 | 49 | 60 | 43 | 37 | 43 | 49 | 10 | 20 |
| | Y | 40 | 30 | 70 | 20 | 30 | 50 | 72 | 60 | 45 | 25 |

- b. Find the extremal of $\int_{x_1}^{x_2} \frac{1+y^2}{y'^2} dx$
- Obtain Taylor and Laurent series expansion about z=0 of function c. $f(z) = \frac{z-1}{z^2 - 2z - 3}$ indicating regions of convergence
- Q. 3 a. A continuous random variable has probability density function as $f(x) = kx^2$ $0 \le x \le 2$, find k, mean and P(0.2 < x < 0.5)
 - b. $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-2)(z-3)} dz$ where C is the circle i. |z|=1 ii. |z|=4
 - c. Reduce the quadratic form to canonical form, find it's rank and signature $21x_1^2+11x_2^2+2x_3^2-30 x_1 x_2+12 x_1 x_3-8 x_3 x_2$

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- Q 4

 a. Using Gram-Schmidt process, construct, an orthonormal basis of (1, 1, 1), (-1, 1, 0) and (1, 2, 1) in R³ have Euclidian inner product Find the probability that at most 4 defective bulbs will be found in a box of 200 bulbs, if it is known that 2% of the bulbs are defective
 - c. By Rayleigh -Ritz method, Solve the boundary value problem y''+y+x=0 0 < x < 1 y(0)=y(1)=0
- Q 5 Ten students got the following percentage of marks in mathematics and statistics

| a. | Maths | 78 | 36 | 98 | ×25 | 75 | -82 | 90 | 62 | 65 | 39 |
|----|-------|----|----|----|-----|----|-----|----|----|----|----|
| Ś | Stats | 84 | 51 | 91 | 60 | 68 | 62 | 86 | 58 | 53 | 47 |

Calculate the coefficient of correlation.

- b. In a normal distribution 7% of the items are below 35 and 89% of the items are below 63. Find the mean and standard deviation
- c. Find the extremal of $\int_{x_1}^{x_2} (y''^2 y^2 + x^2) dx$ 8
- **Q 6** a. Verify Cauchy -Schwartz inequality for u=(1, 2, 4) and v=(-3, 2, 5) 6

 - c. Find Singular value decomposition of $\begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$
