

Time: 3 hour

Max. Marks: 80

Note: 1) Question 1 is compulsory.

2) Attempt any 3 questions from Question 2 to Question 6

3) Figures to the right indicate full marks.

Q1	Attempt All questions	Marks
a)	If $A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & 1 \end{bmatrix}$ then find the eigen values of A^3	5
b)	Find Laplace transform of $f(t) = te^t \cos 2t$	5
c)	Find the Fourier Series for $f(x) = x^2$, where $x \in (-\pi, \pi)$	5
d)	Determine the constant a, b, c, d if $f(z) = x^2 + 2axy + by^2 + i(dx^2 + 2cxy + y^2)$ is analytic.	5
Q2		
a)	A vector field $\vec{F}$ is given by $\vec{F} = (y \sin z - \sin x)i + (x \sin z + 2yz)j + (xy \cos z + y^2)k$ Prove that $\vec{F}$ is irrotational.	6
b)	Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$	6
c)	Show that the function $u = \sin x \cosh y + 2 \cos x \sinh y + x^2 - y^2 + 4xy$ satisfies Laplace's equation, also find analytic function.	8
Q3		
a)	If $\vec{F} = xye^{2z}i + xy^2 \cos zj + x^2 \cos xyk$ find $\text{div} \vec{F}$ and $\text{curl} \vec{F}$	6
b)	Find an analytic function whose real part is $u = y^3 - 3x^2y$. Also find the corresponding imaginary part.	6
c)	Show that the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ is diagonalizable and hence find the transforming matrix and diagonal matrix.	8

Q4

- a) Find $\nabla\phi$ at point $(1, -2, -1)$, where $\phi = 4xz^2 + x^2yz$ 6
- b) Evaluate $\int_0^\infty e^{-2t} \sin^3 t \, dt$, using Laplace transforms 6
- c) Using Partial Fraction method find $L^{-1} \left[\frac{s}{(s^2+1)(s^2+4)(s^2+9)} \right]$ 8

Q5

- a) Find $L \{ t \sqrt{1 + \sin t} \}$ 6
- b) Consider the vector field $\vec{F}$ on $\mathbb{R}^3$ defined by
 $\vec{F}(x, y, z) = y \hat{i} + (z \cos(yz) + x) \hat{j} + (y \cos(yz)) \hat{k}$
 Show that $\vec{F}$ is conservative. 6
- c) Find the Fourier Series for $f(x)$ in $(-\pi, \pi)$ where 8
- $$f(x) = 1 + \frac{2x}{\pi} \quad -\pi \leq x \leq 0$$
- $$= 1 - \frac{2x}{\pi} \quad 0 \leq x \leq \pi$$
- Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

Q6

- a) Obtain Fourier series expansion of $f(x) = 9 - x^2$ in $(0, 2\pi)$ 6
- b) Find Eigen values and Eigen vectors of 6
- $$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$$
- c) 4
- i) Find $L^{-1} \left\{ \log \left(\sqrt{\frac{(s+a)}{(s+b)}} \right) \right\}$
- ii) Find $L^{-1} \left\{ \frac{1}{s^2 - 2s + 5} \right\}$ 4