

(3 Hours)

[Total Marks: 80]

N.B. : 1) Question No. 1 is Compulsory.

2) Answer any THREE questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

Q 1. a) Evaluate the Laplace transform of  $L[(\sin 2t - \cos 2t)^2]$  [5]

b) Determine the constants a, b, c, d so that the function  $f(z) = x^2 + axy + by^2 + i(cx^2 + dxy + y^2)$  is analytic [5]

c) If  $\phi = 3x^2y - y^3z^2$  find  $\nabla \phi$  at the point P (1, -2, -1) [5]

d) Obtain half range sine series for  $f(x) = x^2$  in  $0 < x < 3$  [5]

Q 2. a) Construct analytic function whose real part is  $e^x \cos y$  [6]

b) Find the Fourier series for  $f(x) = |x|$  in  $(-2, 2)$ . [6]

c) Find the Laplace transform of the following

i)  $L[t\sqrt{1 + \sin t}]$  ii)  $L\left\{\frac{\sin t \sin 5t}{t}\right\}$  [8]

Q 3. a) Prove that  $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$  [6]

b) Evaluate inverse Laplace transform using Convolution Theorem  $L^{-1}\left[\frac{s}{(s^2 + a^2)^2}\right]$  [6]

c) Show that the vector field  $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + (3xz^2 + 2z)\hat{k}$  is conservative and find  $\phi(x, y, z)$  such that  $\vec{F} = \nabla \phi$ . [8]

Q 4 a) Find bilinear transformation which maps the points  $z=0, i, -2i$  of  $z$  plane onto the points

$w = -4i, \infty, 0$  of  $w$  plane [6]

b) Prove that  $f_1(x) = 1, f_2(x) = x, f_3(x) = \frac{3x^2 - 1}{2}$  are orthogonal over  $(-1, 1)$ . [6]

c) Find the Fourier transform of  $f(t) = e^{-|t+1|}$  [8]

Q 5 a) Solve Using Laplace transform  $\frac{d^2 y}{dt^2} - 4y = 3e^t$  where  $y(0) = 0$  &  $y'(0) = 3$  [6]

b) Find Complex form of the Fourier series for  $f(x) = e^{ax}$  in  $-\pi < x < \pi$  [6]

c) Verify Green's Theorem for  $\oint_C 2y^2 dx + 3xy dy$  where C is the boundary of the closed region

bounded by  $y = x^2$  and  $y = x$ . [8]

Q 6. a) Evaluate  $L^{-1} \left[ \frac{se^{-\frac{s}{2}} + \pi e^{-s}}{(s^2 + \pi^2)} \right]$  [6]

b) Find the map of the line  $x-y=1$  by transformation  $w = \frac{1}{z}$  [6]

c) Using Stoke's theorem evaluate  $\oint_C (y dx + z dy + x dz)$  where C is the curve of intersection of the sphere  $x^2 + y^2 + z^2 = a^2$  and plane  $x + z = a$  [8]

\*\*\*\*\*