**(Time : 3 hours)** **[Total marks: 80**

**Note : - (1) Question No. 1 is compulsory.**

**(2) Answer any three question from Q 2 to Q 6.**

**(3)Figures to the right indicate full marks .**

1(a) Find the Laplace Transform of $e^t \sin 2t \sin 3t$ . 05

1(b) Construct an analytic function whose imaginary part is $v = \cos x \sinh y$ . 05

1( c ) Find Eigen values of $A^2 - 2A + I$ where $A = \begin{bmatrix} 2 & 1 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & 3 \end{bmatrix}$. 05

1(d) Find the Fourier Series Expansion $f(x) = x$ ,where $x \in (-\pi, \pi)$ 05

2(a) Find the direction derivative of $\phi(x, y, z) = \sin(xy) + e^{3xz}$ in the direction of the vector $v = i - 2j + 2k$ at the point $P = \left(1, \frac{\pi}{4}, 1\right)$ 06

2(b) Find Fourier series of $f(x) = x(\pi - x), 0 < x < \pi$ . 06

2(c ) Find Inverse Laplace Transform of (i) $\frac{2s+3}{s^2+2s+2}$ (ii) $\frac{s+2}{s^2(s+3)}$ . 08

3(a) Find Eigen Values and Eigen Vector of the following matrix 06

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$$

3(b) Find orthogonal trajectory of family of curve $3x^2y - y^3 = c$ . 06

3(c ) Find the Fourier Series for $f(x)$ in $(0,2\pi)$where

$$f(x) = \begin{cases} x & ,0 < x \le \pi \\ 2\pi - x & ,\pi \le x < 2\pi \end{cases}$$

Hence deduce that

$$\sum_{n \in Odd\ natural\ numbers} \frac{1}{n^4} = \frac{\pi^4}{96}$$ 08

4(a) Prove that is $\bar{F} = \frac{xi+yj}{x^2+y^2}$ is both solenoidal and irrotational . 06

4(b) Evaluate $\int_0^\infty e^{-2t} t \cos t \ dt$. 06

4(c) Show that the matrix

$A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ diagonalizable and find transforming matrix and Diagonal matrix . 08

5(a) Find the inverse Laplace Transform of $\frac{(s+2)^2}{(s^2+4s+8)^2}$ by using convolution theorem. 06

5(b) Construct an analytic function $f(z) = u + iv$, where 06

$u - v = (x - y)(x^2 + 4xy + y^2)$ .

5(c) Evaluate by using Green's theorem

$\int_C (e^{x^2} - xy)dx + (y^2 - ax)dy$, where C is the circle $x^2 + y^2 = a^2$ . 08

6(a) By using CHT theorem and find $A^{-1}$ and $A^4$ . where $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ 06

6(b) Obtain half range sine series in $(0, \pi)$ for $f(x) = x(\pi - x)$,

Hence show that

$$\frac{\pi^3}{32} = 1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots$$ 06

6(c) Evaluate $\int_0^\infty e^{-2t} \left( \int_0^t \frac{e^{-u} \sin 2u}{u} du \right) dt$ 08

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