Paper / Subject Code: 51221 / Engineering Mathematics-III

[Total marks: 8 (Time : 3 hours) (1) Question No. 1 is compulsory. (2) Answer any three question from Q 2 to Q 6. (3) Figures to the right indicate full marks. (a) Find the Laplace Transform of $e^t \sin 2t \sin 3t$ 1(b) Construct an analytic function whose imaginary part is $v = \cos x \sinh y$ 1(c) Find Eigen values of $A^2 - 2A + I$ where $A = \begin{bmatrix} 2 & 1 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & 3 \end{bmatrix}$ 1(d) Find the Fourier Series Expansion f(x) = x, where $x \in (-\pi, \pi)$ 2(a) Find the direction derivative of $\phi(x, y, z) = \sin(xy) + e^{3xz}$ in the direction of the vector v = i - 2j + 2k at the point $P = \left(1, \frac{\pi}{4}, 1\right)$ 2(b) Find Fourier series of $f(x) = x(\pi - x)$, $0 < x < \pi$.

2(c) Find Inverse Laplace Transform of (i)
$$\frac{2s+3}{s^2+2s+2}$$
 (ii) $\frac{s+2}{s^2(s+3)}$.

3(a) Find Eigen Values and Eigen Vector of the following matrix

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$$

3(b) Find orthogonal trajectory of family of curve $3x^2y - y^3 =$

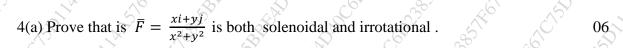
B(c) Find the Fourier Series for f(x) in (0.2π) where

$$f(x) = \begin{cases} x, & 0 < x \le \pi \\ 2\pi - x, & \pi \le x < 2\pi \end{cases}$$

Hence deduce that

$$\sum_{n \in Odd \ natural \ numbers} \frac{1}{n^4} = \frac{\pi^4}{96}$$

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4(b) Evaluate
$$\int_0^\infty e^{-2t} t \cos t \ dt$$
.

4(c) Show that the matrix

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$$A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$
 diagonalizable and find transforming matrix and Diagonal matrix. 08
$$5(a) \text{ Find the inverse Laplace Transform of } \frac{(s+2)^2}{(s^2+4s+8)^2} \text{ by using convolution theorem.}$$
06
$$5(b) \text{ Construct an analytic function } f(z) = u + iv, \text{ where}$$
06

$$u - v = (x - y)(x^2 + 4xy + y^2).$$

5(c) Evaluate by using Green's theorem

$$\int_{C} (e^{x^{2}} - xy) dx + (y^{2} - ax) dy, \text{ where C is the circle } x^{2} + y^{2} = a^{2}.$$

6(a) By using CHT theorem and find
$$A^{-1}$$
 and A^{4} , where $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ 06
6(b) Obtain half range sine series in $(0,\pi)$ for $f(x) = x(\pi - x)$,

Hence show that
$$\frac{\pi^{3}}{32} = 1 - \frac{1}{3^{3}} + \frac{1}{5^{3}} - \frac{1}{7^{3}} + \cdots$$

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6(c) Evaluate
$$\int_0^\infty e^{-2t} \left(\int_0^t \frac{e^{-u} \sin 2u}{u} du \right) dt$$