Q. P. Code : 37078

(3 Hours) **[Total marks : 80**

Note : -
1) Question number **1** is **compulsory.**
2) Attempt any **three** questions from the remaining **five** questions.
3) **Figures** to the **right** indicate **full** marks.

Q.1 a) Evaluate $\int_0^\infty e^{-2t} sin^2 2t \, dt$. 05

b) Find an analytic function $f(z) = u + iv$ where $u + v = e^x(\cos y + \sin y)$. 05

c) Obtain Fourier series of $x \cos x$ in $(-\pi, \pi)$. 05

d) Evaluate $\int_C \bar{F} \cdot d\bar{r}$ where $\bar{F} = x^2 \, i + xy \, j$ from $(0,0)$ to $(1,1)$ along the parabola $y^2 = x$. 05

Q.2 a) Find half-range cosine series for $f(x) = e^x, \; 0 < x < 1$. 06

b) Prove that
$$\bar{F} = (x + 2y + az)\, i + (bx - 3y - z)\, j + (4x + cy + 2z)\, k$$
is solenoidal and determine the constants a, b, c if $\bar{F}$ is irrotational. 06

c) Prove that $w = i\left(\frac{z-i}{z+i}\right)$ maps upper half of the z −plane into the interior of the unit circle in the w −plane. 08

Q. 3 a) Prove that $J_n(x)$ is an even function if n is even integer and is an odd function if n is odd integer. 06

b) Find the inverse Laplace transform of $\frac{s^2+2s+3}{(s^2+2s+5)(s^2+2s+2)}$. 06

c) Obtain the complex form of Fourier series for $f(x) = e^{ax}$ in $(0, a)$. 08

Q. 4 a) Prove that $\nabla f(r) = f'(r)\frac{\bar{r}}{r}$ and hence, find f if $\nabla f = 2r^4 \bar{r}$. 06

b) Prove that $4J''_n(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$. 06

c)
(i) Find the Laplace transform of $e^{4t}\ sin^3 t$. 04

(ii) Find the Laplace transform of $t\sqrt{1+\sin t}$. 04

Q. 5 a) Prove that $\int x \cdot J_{\frac{2}{3}}\left(x^{\frac{3}{2}}\right) dx = -\frac{2}{3}\ x^{-\frac{1}{2}} J_{-\frac{1}{3}}\left(x^{\frac{3}{2}}\right)$. 06

b) Find p if $f(z) = r^2 \cos 2\theta + i\, r^2 \sin p\theta$ is analytic. 06

c) If $f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2-x), & 1 \le x \le 2 \end{cases}$ with period 2, show that 08

$f(x) = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \cos(2n+1)\pi x.$

Q. 6 a) Show that the set of functions $\cos nx$, $n = 1, 2, 3, \ldots\ldots$ is orthogonal on $(0, 2\pi)$. 06

b) Use Stoke's theorem to evaluate $\int_C \bar{F} \cdot d\bar{r}$ where $\bar{F} = (2x - y)\, i - yz^2\, j - y^2 z\, k$ and S is the surface of hemisphere $x^2 + y^2 + z^2 = a^2$ lying above the xy −plane. 06

c) Use Laplace transform to solve 08

$\frac{d^2y}{dt^2} + y = t$ with $y(0) = 1, y'(0) = 0.$

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