(3 hours) [Total Marks: 80]

Note:

1. Question-1 is compulsory.
2. Answer any three questions from remaining five.
3. Assume suitable data if necessary.
4. Numbers in the right indicate marks.

1. Answer the following questions. (Each question carry 5 marks) 20

(a) Describe the data holding in digital systems and derive the transfer function of zero-order hold (ZOH).

(b) Draw the frequency spectrum of the ideal low-pass filter and comment on its realizability with justification.

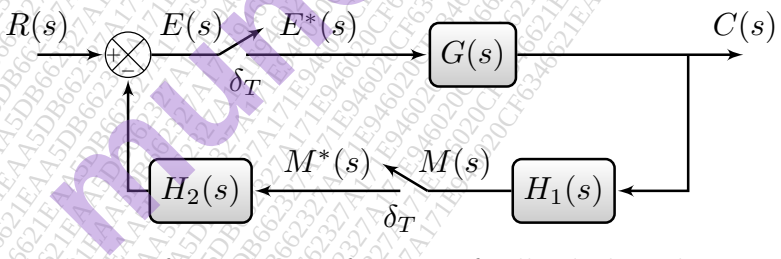
(c) Explain quantization in digital control systems and derive the bound on the quantization error.

(d) Clearly explain the distinction between detectability and observability for discrete-time systems.

2. (a) Describe bilinear transformation approach for discretization of continuous-time systems in detail. Also, comment on the mapping between s-plane and z-plane under such discretization. 10

(b) Discretize continuous-time PID controller using trapezoidal rule to approximate integral term and two-point difference to approximate derivative term. 10

3. (a) Derive the closed-loop pulse transfer function $\frac{C(s)}{R(s)}$ for the digital control system shown in figure below. Assume sampling time to be 1 second. 10



(b) Make a rough sketch of root locus of a unity feedback digital control system whose open loop transfer function is given as 10

$$G(z) = \frac{Kz(1-e^{T})}{(z-1)(z-e^{T})}$$

Also determine the critical value of K. Take sampling frequency $f_s = 1$Hz.

4. (a) Design the state feedback controller $u[k] = Kx[k] + k_0 r[k]$ such that the steady-state error of the closed loop is zero for the unit step input. 10

$$x[k+1] = \begin{bmatrix} -0.6 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & -0.8 \end{bmatrix} x[k] + \begin{bmatrix} 2 \\ 0.6 \\ 1 \end{bmatrix} u[k]$$

(b) Represent the discrete-time system of **Q-4(a)** in controllable canonical form and observable canonical form using similarity transforms. 10

5. (a) Discretize the continuous time state-space equation $\dot{x} = Ax + Bu$ and obtain the discrete-time state-space representation. 10

(b) Design a deadbeat controller for the following discrete-time system. 10

$$x[k+1] = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} x[k] + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u[k]$$

6. Answer any three of the following questions. 20

(a) Derive the mapping between s-plane and z-plane when discretizing a continuous-time system using impulse invariance approach. Determine the region of stability on z-plane under this mapping.

(b) Determine the stability of a discrete-time system which has characteristic equation $z^4 + 1 = 0$.

(c) Explain the distinction between observability and detectability.
