

(3 Hours)

Total Marks: 80

- Note: 1) Question no 1 is compulsory.
 2) Attempt any 3 question out of remaining.
 3) Each question carries 20 Marks.
 4) Figures to right indicate full marks.

- Q.1 a) Calculate the coefficient of correlation between x and y from the following data: [5]
 $N=10, \sum x = 140, \sum y = 150, \sum (x - 10)^2 = 180, \sum (y - 15)^2 = 215$ and
 $\sum (x - 10)(y - 15) = 60$
- b) Evaluate $\oint_c \log z dz$ where c is the circle with centre at origin and radius 1. [5]
- c) Find the projection of $u = (3, 0, 4)$ along and perpendicular to $v = (2, 3, 3)$ [5]
- d) Find the eigen values of $3A^2 - 2A + 5I$ where $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{bmatrix}$ [5]
- Q.2 a) Find the extremals of $\int_{x_1}^{x_2} (1 + x^2 y') y' dx$ [6]
- b) Using Gram-Schmidt process, transform the basis $\{v_1, v_2, v_3\}$ into orthogonal basis [6]
 where $v_1 = (1, 0, 0), v_2 = (3, 7, -2), v_3 = (0, 4, 1)$.
- c) Show that $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & -2 \\ 0 & -6 & -3 \end{bmatrix}$ is diagonalisable and hence find the transforming matrix [8]
 and diagonal form of A.
- Q.3 a) For a normal variable x, with mean 10 and standard deviation 4, find (i) $P(|x-14| < 1)$ [6]
 and (ii) $P(x \leq 12)$
- b) Fit a binomial distribution for the following data [6]
 $x: 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6$
 Frequency: 5 18 28 12 7 6 4
- c) Using Rayleigh-Ritz Method find the solution of $I = \int_0^1 (2xy - y^2 - y'^2) dx$ where [8]
 $0 \leq x \leq 1$ and $y(0)=y(1)=0$.
- Q.4 a) Find the lines of regression for following data [6]
 $x: 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11$
 $y: 11 \ 14 \ 14 \ 15 \ 12 \ 17 \ 16$
- b) If $f(\alpha) = \oint_C \frac{3z^2 - z + 5}{z - \alpha} dz$ where C is the circle $|z| = 3$ then find $f(1), f(-1), f(-i)$, [6]
- c) Check whether the set of pairs of real numbers of the form $(1, u)$ with operations [8]
 $(1, u) + (1, v) = (1, u + v)$ and $k(1, u) = (1, ku)$ is a vector space.
- Q.5 a) Find the value of k such that $f(x) = \begin{cases} k(1 - x^2) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$ is a probability function [6]
 and hence find $P(0.1 < x < 0.2)$ and $P(x > 0.5)$
- b) If $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$ then show that $3 \cdot \tan A = A \cdot \tan 3$ [6]
- c) Find all possible expansions of $f(z) = \frac{1}{(z-1)(z-2)}$. [8]
- Q.6 a) Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5 + 4\cos\theta} d\theta$ using Cauchy Residue Theorem. [6]
- b) Show that the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ is non-derogatory. [6]
- c) Find the m.g.f. of Poisson's Distribution about origin. Hence find its mean and variance [8]