Time: 3 hrs. **Max. Marks: 80**

N.B. : 1. Q1 is compulsory

2. Attempt any three questions from Q2 to Q6.

3. Figures to the right indicate full marks.

Q1. (a) Evaluate the integral $\int\limits_{C} \dfrac{z^2}{(z-i)(z+2)^2}\ dz$, C: $|z-i|=1$.   **5**

(b) A r.v. X has the distribution   **5**

x:	0	1	2	3	4	5	6	7
p(x):	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find i) k ii) evaluate $P(X<6)$

(c) Find the usual inner product between the two vectors $(2,1,-3)$ **5**

and $(-1,1,2)$. Find the norm of each vectors and verify the Cauchy

Schwarz inequality.

(d) The given data indicates weight x and heights y of 1000 men. $\overline{x}=150\,\text{lbs}$,   **5**
$\overline{y}=68\,\text{inches}$, $\sigma_x=20\,\text{lbs}$, $\sigma_y=2.5\,\text{inches}$, $r=0.6$. John weighs 200 lbs.

Find the line of regression of y on x and estimate the height of John.

Q2. (a) Find the Extremal of $\int\limits_{x_1}^{x_2} \sqrt{1+(y')^2}\ dx$.   **6**

(b) Find the Laurent series expansion of $\dfrac{z+2}{z^2-1}$ convergent in the   **6**

domain $|z|>1$.

(c) Reduce the quadratic form $x_1^2+2x_2^2+2x_3^2-2x_1x_2+x_1x_3-2x_2x_3$ **8**

to diagonal form by congruent transformation. Obtain the transformation

applied in the reduction and Find the rank, index and class value.

Q3. (a) Find the Extremal of $\int_0^1 y\,y' + \left(y''\right)^2\ dx,$ **6**

$y(0) = 0,\ y'(0) = 1,\ y(1) = 2,\ y'(1) = 4$

(b) From a vessel containing 3 white and 5 black balls, 4 balls are transferred **6**

into an empty vessel. From this vessel a ball is drawn and found to be white.

Find the probability that out of four balls transferred 3 are white and 1 is black.

(c) Find a singular value decomposition of the matrix $\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$. **8**

Q4. (a) Evaluate the integral $\int_C \dfrac{\sin^2 z}{z^3}\ dz,\ \ C:|z| = 1,$ using Cauchy **6**

integral formula.

(b) Using Gram Schmidt method, find an orthogonal set of vectors **6**

corresponding to $(1,1,0,1),\ (-1,0,1,0),\ (0,0,1,-1)$.

(c) After correcting 50 pages of the proof of a book, the proof reader finds that **8**

there are on the average 2 errors per 5 pages. How many pages would one expect

to find with 0, 1, 2, 3 errors in 1000 pages of the first print of the book.

Q5. (a) Evaluate the Integral $\int_C \overline{z}\ dz$ along astraight line **6**

from $z = 0$ to $z = 4 + 2i$.

(b) Find the rank correlation coefficient for the following data. **6**

x : 10 12 18 16 15 40
y : 12 18 20 15 50 25

(c) Using Rayleigh-Ritz method, find an approximate solution for the **8**

Extremal of $\int_0^1 \left(y'\right)^2 - 4y^2 + 2x^2 y\ dx,\ y(0) = 0,\ y(1) = 0$

Q6. (a) If $f(x) = \begin{cases} \frac{x}{2} & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$ is a pdf of a random variable X, then **6**

find $E(X), \ \text{var}(X), \ \text{var}(3X)$.

(b) Let $W_1 = \{(x, y) \mid x, y \text{ are real numbers}, \ y = m x\}$ and **6**

$W_2 = \{(x, y) \mid x, y \text{ are real numbers}, \ x y \geq 0\}$. Show that

W_1 is a subspace and W_2 is not a subspaces of two dimensional space.

(c) A Chemical Engineer is investigating the effect of process operating **8**

temperature x on product yield y. The study results in the following data

x : 100 110 120 130 140 150 160 170 180 190
y : 45 51 54 61 66 70 74 78 85 89

Find the equation of the least square lines which will enable us to predict

(i) yield on the basis of temperature (ii) temperature on the basis of yield.