

(3 Hours)

[Total Marks: 80]

- Note: 1) Question no 1 is compulsory.
 2) Attempt any 3 question out of remaining.
 3) Each question carries 20 Marks.
 4) Figures to right indicate full marks.

- Q.1 a) Compute the Spearman's Rank correlation coefficient for the following data: [5]
 $x: 18 \quad 20 \quad 34 \quad 52 \quad 12$
 $y: 39 \quad 23 \quad 35 \quad 18 \quad 46$
- b) Evaluate $\int_0^{2+i} z^2 dz$ along the line $x=2y$. [5]
- c) Find the projection of $u = (3, 1, 3)$ along and perpendicular to $v = (4, -2, 2)$ [5]
- d) Find the eigen values of $5A^2 - 6A + I$ where $A = \begin{bmatrix} -1 & 5 & 9 \\ 0 & -3 & 4 \\ 0 & 0 & 2 \end{bmatrix}$ [5]
- Q.2 a) Find the extremals of $\int_{x_1}^{x_2} \frac{y'^2}{x^2} dx$ [6]
- b) Use Gram-Schmidt process to construct the orthogonal basis from $x_1 = (1, 1, 1)$, $x_2 = (0, 1, 1)$, $x_3 = (0, 0, 1)$. [6]
- c) Show that $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ is diagonalisable and hence find the transforming matrix and diagonal form of A. [8]
- Q.3 a) For a normal variable x , with mean 2.5 and standard deviation 3.5, find the probability that (i) $2 \leq x \leq 4.5$ and (ii) $-1.5 \leq x \leq 5.5$ [6]
- b) The ratio of the probability of 3 successes in 5 independent trials to the probability of 2 successes in 5 independent trials is 1:4. What is the probability of 4 successes in 6 independent trials? [6]
- c) Using Rayleigh-Ritz Method find the solution of $I = \int_0^1 (2xy + y^2 - y'^2) dx$ where $0 \leq x \leq 1$ and $y(0)=y(1)=0$. [8]
- Q.4 a) Find the line of regression of Y on X for following data [6]
 $x: 10 \quad 12 \quad 13 \quad 16 \quad 17 \quad 20 \quad 25$
 $y: 19 \quad 22 \quad 24 \quad 27 \quad 29 \quad 33 \quad 37$. Hence find the value of y at $x=15.5$
- b) Evaluate $\oint_C \frac{3z^2+2z-2}{(z-1)(z-2)} dz$ where C is the curve (i) $|z| = \frac{1}{2}$, (ii) $|z| = \frac{3}{2}$, (iii) $|z| = 3$ [6]
- c) Find the m.g.f. of Poisson's Distribution about origin. Hence find its mean and variance [8]
- Q.5 a) If x is a continuous random variable with probability distribution function [6]
 $f(x) = \begin{cases} \frac{x}{6} + k & \text{if } 0 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$ then find the value of k and $P(1 \leq x \leq 2)$
- b) If $A = \begin{bmatrix} 3 & 1 \\ 2 & 2 \\ 1 & 3 \\ 2 & 2 \end{bmatrix}$ then find the values of matrices e^A and 4^A . [6]
- c) Find all possible expansions of $f(z) = \frac{2-z^2}{z(1-z)(2-z)}$. [8]
- Q.6 a) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+3\sin\theta}$ using Cauchy Residue Theorem. [6]
- b) Show that the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$ is derogatory and find its minimal polynomial [6]
 also.
- c) Show that the set of real numbers is a vector space with the operations defined as $x + y = xy$ be addition and $cx = x^c$ be scalar multiplication. [8]