

(3 Hours)

(Total Marks : 80)

- N.B.:** 1) **Question No. 1 is Compulsory.**
2) Attempt **any three** from the **remaining**.

1. a) Find the extremal of $\int_{x_0}^{x_1} \frac{1+y^2}{y'^2} dx$. (05)
- b) Is the following set of vectors in P_2 linearly independent? $2 - x + 4x^2$, $3 + 6x + 2x^2$, $2 + 10x - 4x^2$? (05)
- c) Show that Eigen values of Hermitian matrix are real. (05)
- d) Evaluate $\int (z^2 - 2\bar{z} + 1) dz$ over a closed circle $x^2 + y^2 = 2$. (05)
2. a) Find the extremal $\int_0^\pi (y^2 - y'^2 - 2y \cos x) dx$, $y(0) = 0$, $y(\pi/2) = 0$. (06)
- b) Find the Eigen Values and Eigen Vectors of the matrix $A^3 + 3I$, where

$$A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$
 (06)
- c) Obtain all possible expansion of $f(z) = \frac{z}{(z-1)(z-2)}$ about $z = -2$ indicating region of convergence. (08)
3. a) Verify Cayley - Hamilton Theorem for $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -2 \\ -2 & 0 & 1 \end{bmatrix}$ and find A^{-1} . (06)
- b) Using Cauchy's Residue Theorem evaluate $\int_C \frac{e^z}{z^2 + \pi^2} dz$ where C is $|z|=4$. (06)
- c) Show that a closed curve 'C' of a given fixed length (perimeter) which encloses maximum area is a circle. (08)
4. a) Find an orthonormal basis for the subspace of R^3 by applying Gram-Schmidt process, where $u_1 = (1,0,1,1)$, $u_2 = (-1,0,1,1)$, $u_3 = (0, -1,1,1)$. (06)
- b) Find A^{20} for the matrix $A = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$. (06)
- c) Reduce the Quadratic Form $2xy + 2yz + 2zx$ to diagonal form by orthogonal reduction method. (08)
5. a) Using Rayleigh-Ritz Method, find an approximate solution to the extremal problem $\int_0^1 (y'^2 - y^2 - 2yx) dx$, $y(0) = 0$, $y(1) = 0$. (06)
- b) Let V be a vector space containing 2×2 matrices and $W \subseteq V$ such that $W = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$. Is W a subspace of V ? Justify. (06)
- c) Show that the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalizable. Also find the transforming matrix and diagonal matrix. (08)
6. a) Using Cauchy's Residue Theorem, evaluate $\int_0^{2\pi} \frac{d\theta}{13+5 \sin \theta}$. (06)
- b) Evaluate $\int_{1-i}^{2+i} (2x + 1 + iy) dz$ along the curve $x = t + 1$, $y = 2t^2 - 1$. (06)
- c) Find the singular value decomposition of the matrix $A = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$ (08)