Q. P. Code: 38424

Time: 3 hours Total marks: 80

N.B. (1) Question No.1 is compulsory.
(2) Answer any three questions from remaining .
(3) Figures to the right indicate full marks.

Q1.

a) Find the extremal of $\int_{x_0}^{x_1} \frac{1+y^2}{y'^2} dx$ 05

b) Evaluate $\int_C \frac{\sin^2 z}{\left(z-\frac{\pi}{6}\right)^3} dz$, where C is the circle $|Z|=1$ 05

c) If $A=\begin{bmatrix} \pi & \frac{\pi}{4} \\ 0 & \frac{\pi}{2} \end{bmatrix}$ find Cos A 05

d) The number of messages sent per hour over a computer network has the following probability distribution 05

x	10	11	12	13	14	15
P(X=x)	.08	3k	6k	4k	4k	.07

Find the mean and variance of number of messages sent per hour.

Q2.

a) Construct an Orthonormal Basis of R^3 using Gram Schmidt process to S = $\{(1,0,0),(3,7,-2),(0,4,1)\}$ 06

b) Evaluate $\int_0^{2+i} (\bar{z})^2 dz$ along 06

i) $y=\frac{x}{2}$

ii) The real axis to 2 and then vertically to 2+i

c) i) An underground mine has 5 pumps installed for pumping out storm water.The probability of any one of the pumps failing during the storm is $\frac{1}{8}$.What is the probability that at least 2 pumps will be working. 04

ii) Let W be the set of 2x2 matrices of the form $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ 04

Show that W is a subspace of space V of all 2x2 matrices

Q3.(a) Calculate Karl Pearson's coefficient of correlation between expenditure and sales from data given

AdvertisingExpenses('000 Rs)	39	65	62	90	82	75	25	98	36	78
Sales(Lakhs of Rupees)	47	53	58	86	62	68	60	91	51	84

06

b) Show that the matrix A= $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ is derogatory 06

c) Evaluate $\int_0^{2\pi} \frac{d\theta}{13+12\cos\theta}$ 08

Q4 .

a) Using Cauchy's Residue Theorem evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)} dz$ 06

where C is the circle $|z| = 3$

b) Find the extremals of the functional $\int_{x_0}^{x_1} (y''^2 - y^2 + x^2) dx$ 06

c) i) Assume that the probability of an individual coal miner being injured in a mine accident during a year is $\frac{1}{2400}$ .Calculate the probability that in a mine employing 200 miners there will be at least one fatal accident in a year 04

ii) If X denotes the outcome when a fair die is tossed ,Find the M.G.F of X about the origin .Hence find the first two moments about the origin.

04

a) The IQ's of army volunteers in a given year are normally distributed with

Q5 mean 110 and standard deviation 10.The army wants to give advanced training to 20% of those recruits with the highest scores. What is the lowest IQ score acceptable for advanced training. 06

b) Solve by Rayleigh Ritz method the boundary value problem

$I = \int_0^1 \left(y'^2 - y^2 - 2xy \right) dx$ given y(0)=0 and y(1)=0 06

c) Show that the matrix $A=\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$ is similar to a diagonal matrix. 08

Find the transforming matrix and the diagonal matrix.

Q6. (a) Verify Cayley Hamilton Theorem for $A=\begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$ 06

. Hence find A^{-1}

(b) Obtain Taylor's and Laurent's series expression for $f(z) = \dfrac{z-1}{\left(z^2 - 2z - 3\right)}$ 06

indicating region of convergence.

(c) i) The lines of regression of bivariate population are 8x-10y+66=0 and 40x-18y=214 .The variance of x is 9. Find

a) coefficient of correlation r

b) the standard deviation of y 04

ii) If a,b,c are three positive numbers then using Cauchy Schwarz inequality prove that $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 3^2$ 04

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