(Time : 3 Hours) **[Total marks: 80**

Note: **1). Question 1 is compulsory.**

2) Attempt any 3 questions from Question 2 to Question 6

Q1	**Attempt All questions**		**Marks**
A	If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$ then find the eigen values for the matrix $A^3 + 5A + 8I + A^{-1}$		**5**
B	Find Laplace transform of $f(t) = te^{-t}\sin(4t)$		**5**
C	Find the Fourier Series Expansion $f(x) = x$,where $x \in (-\pi, \pi)$		**5**
D	Determine the constant a,b,c,d if $f(z) = x^2 + 2axy + by^2 + i(dx^2 + 2cxy + y^2)$ is analytic.		**5**
Q2			
A	Using Green's theorem in a plane to evaluate the line integral $\oint_C (xy^2 - y)dx + (x + y^2)dy$ Where C is the triangle with vertices at (0,0), (2,0) and (2,2) and it is traversed in anticlockwise direction		**6**
B	Find the matrix $A_{2\times2}$ whose eigen values are 4 and 1 and their corresponding eigen vectors are $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$		**6**
C	Find the analytic function $f(z) = u + iv$ such that $u - v = \dfrac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$ $when\ f\left(\frac{\pi}{2}\right) = 0$		**8**
Q3			
A	Find the direction derivative of $\phi(x, y, z) = \sin(xy) + e^{3xz}$ in the direction of the vector $v = \boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}$ at the point $P = \left(1, \frac{\pi}{4}, 1\right)$		**6**
B	Find an analytic function $f(z)$ whose real part is given $u(x, y) = x^3 - 3xy^2 + 2x + y$		**6**
C	Find the Eigen values and Eigen vectors of $A = \begin{bmatrix} \frac{37}{60} & \frac{17}{60} & \frac{17}{60} \\ \frac{1}{5} & \frac{7}{10} & \frac{1}{5} \\ \frac{1}{12} & -\frac{1}{12} & \frac{5}{12} \end{bmatrix}$ And show that it is diagonalizable matrix and find its transforming matrix and the diagonal form		**8**

Q4

A Using Stokes theorem to evaluate $\int_C \bar{F} \cdot d\bar{r}$ — **6**
Where $\bar{F} = (x - y - z)i + (y - z - x)j + (z - x - y)k$ over the paraboloid $x^2 + y^2 = 4 - z\,, z \geq 0$

B Find the orthogonal trajectories of family of curves given by $x^3y - xy^3 = c$ — **6**

C Using Convolution theorem, find the inverse Laplace transform of — **8**

$$\phi(s) = \frac{s+1}{(s^2+2s+2)(s^2+2s+5)}$$

Q5

A Evaluate $\int_0^\infty \frac{\cos 6t - \cos 4t}{t}\, dt$, using Laplace transforms — **6**

B Consider the vector field $\bar{F}$ on $\mathbb{R}^3$ defined by — **6**

$$\bar{F}(x,y,z) = y\,\hat{\boldsymbol{\imath}} + (z\cos(yz) + x)\,\hat{\boldsymbol{\jmath}} + (y\cos(yz))\,\hat{\boldsymbol{k}}$$

Show that $\bar{F}$ is conservative and find its scalar potential.

C Find the Fourier Series for $f(x)$ in $(0,2\pi)$ where — **8**

$$f(x) = \begin{cases} x & ,0 < x \leq \pi \\ 2\pi - x & ,\pi \leq x < 2\pi \end{cases}$$

Hence deduce that

$$\sum_{n \in Odd\ natural\ numbers} \frac{1}{n^4} = \frac{\pi^4}{96}$$

Q6

A Obtain half range sine series in $(0,\pi)$ for $f(x) = x(\pi - x)$, — **6**
Hence show that

$$\frac{\pi^3}{32} = 1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots$$

B Using Cayley Hamilton theorem find — **6**

$$A^6 - 12A^5 + 30A^4 + 72A^3 - 207A^2 - 110A + 330I$$

Where $A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

C

i) Find $L^{-1}\left\{\log\left(\sqrt{\frac{s^2+a^2}{s^2}}\right)\right\}$ — **4**

ii) Find $L^{-1}\left\{\frac{s-1}{(2s+1)^2}\right\}$ — **4**
