

(3 Hours)

[Total marks : 80

- Note** : -
- 1) Question number 1 is **compulsory**.
 - 2) Attempt any **three** questions from the remaining **five** questions.
 - 3) **Figures** to the **right** indicate **full marks**.

- Q.1 a) Find the Laplace transform of $\sinh^5 t$. 05
- b) Find an analytic function whose imaginary part is $e^{-x}(y \cos y - x \sin y)$. 05
- c) Find the Fourier series for $f(x) = 1 - x^2$ in $(-1, 1)$. 05
- d) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = 2x \vec{i} + (xz - y) \vec{j} + 2z \vec{k}$ from $O(0, 0, 0)$ to $P(3, 1, 2)$ along the line OP . 05
- Q.2 a) Find a cosine series of period 2π to represent $\sin x$ in $0 \leq x \leq \pi$. 06
- b) Find a, b, c if $\vec{F} = (axy + bz^3) \vec{i} + (3x^2 - cz) \vec{j} + (3xz^2 - y) \vec{k}$ is irrotational. 06
- c) Find the image of the circle $|z| = k$ where k is real under the bilinear transformation $w = \frac{5-4z}{4z-3}$. 08
- Q.3 a) Prove that $J_{\frac{1}{2}}(x) = \tan x \cdot J_{-\frac{1}{2}}(x)$. 06
- b) Find the inverse Laplace transform of the following function by convolution theorem $\frac{(s+2)^2}{(s^2+4s+8)^2}$. 06
- c) Obtain the complex form of Fourier series for $f(x) = e^{ax}$ in $(-l, l)$ where a is not an integer. 08
- Q.4 a) Find the angle between the normals to the surface $xy = z^2$ at the points $(1, 4, 2)$ and $(-3, -3, 3)$. 06
- b) Prove that $x^2 J_n''(x) = (n^2 - n - x^2) J_n(x) + x J_{n+1}(x);$
 $n = 0, 1, 2, \dots$ 06

- c)
 (i) Find the Laplace transform of $\sinh at \sin at$. 04
 (ii) Find the Laplace transform of $te^{-4t} \sin 3t$. 04

- Q. 5 a) Prove that $J_2(x) = J''_0(x) - \frac{J_0'(x)}{x}$. 06
 b) If $v = e^x \sin y$, show that v is harmonic and find the corresponding analytic function. 06

- c) Find the Fourier series for $f(x)$ in $(0, 2\pi)$, 08

$$f(x) = \begin{cases} x, & 0 < x \leq \pi \\ 2\pi - x, & \pi \leq x < 2\pi \end{cases}$$

Hence, deduce that

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$$

- Q. 6 a) Show that the set of functions $\cos nx$, $n = 1, 2, 3, \dots$ is orthogonal on $(0, 2\pi)$. 06

- b) Using Green's theorem evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is the curve enclosing the region bounded by $y^2 = 4ax$, $x = a$ in the plane $z = 0$ and 06

$$\vec{F} = (2x^2y + 3z^2) \mathbf{i} + (x^2 + 4yz) \mathbf{j} + (2y^2 + 6xz) \mathbf{k}.$$

- c) Use Laplace transform to solve 08

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 1 \text{ with } y(0) = 0, y'(0) = 1.$$
