Q.P. Code: 25012

(3 Hours) *Total Marks :80*

Note: 1) Question No.1 is compulsory
2) Attempt any Three from the remaining

Q1

A) Evaluate using Laplace transform $\int_0^t e^{-\sqrt{2}\,t}\,\frac{sint Sinht}{t}\,dt$ — 5

B) Prove that f(z) = z^n is analytic hence find $f'(z)$ — 5

C) Find a Fourier series to represent f(x) = $\sqrt{1-cosx}$ in (-π , π) . — 5

D) Find f(r) ,so that f(r)$\bar{r}$ is solenoidal — 5

Q2

A) Find analytic function f(z)=u+iv ,if u = e^x(xcosy –ysiny) — 6

B) Find the Bilinear transformation which maps the points z = ∞ , i , 0 onto the points w = 0, i ,∞ — 6

C) Obtain the fourier series for f(x) = $\begin{cases} 2\pi - x, & \pi < x < 2\pi \\ x, & 0 < x < \pi \end{cases}$ — 8

With period Hence deduce that $\frac{\pi^2}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\ldots$

Q3

A) Find inverse Laplace transform of (i) $\log\left(\frac{s^2+a^2}{s^2+b^2}\right)$ (ii) $\frac{e^{-2s}}{s^2+8s+25}$ — 6

B) Find Complex form of Fourier Series of e^{ax} in (-a , a) — 6

C) Verify Greens Theorem for $\int_C (x^2-y)dx + (2y^2+x)dy$ where C is the closed curve of the region bounded by y = 4 and y = x^2 — 8

Q4

A) Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\,.\,sinx$ — 6

B) Use Gauss's Divergence theorem to evaluate $\iint_S \bar{N}.\bar{F}ds$ where $\bar{F}$=x^2i+zj +yzk and S is the surface of the cube bounded by x=0 ,x=1,y=0 ,y=1,z= 0 ,z=1 — 6

C) Solve using Laplace transform(D^2 +2D+5)y =e^{-t} sint ,given y(0)=0 and y′(0)=1 — 8

Q5

A) Find half range sine series for f(x)=x(π-x) in (0 , π) Hence find value of $\sum \frac{(-1)^n}{(2n-1)^3}$ — 6

B) Find the image of |z|<1 under the bilinear transformation w = $\frac{i-z}{z+i}$ also find the fixed point. — 6

c) Prove that y =x^{-n}.J_n(x) is a solution of the equation , — 8

$$x\frac{d^2y}{dx^2} + (1+2n)\frac{dy}{dx} + xy = 0$$

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Q6

A) Find the directional derivative of $\phi = x^2 y\cos z$ at $(1, 2, \frac{\pi}{2})$ in the direction of $(2i + 3j + 2k)$ 6

B) Find inverse Laplace transform of $\frac{1}{(s^2+4s+13)^2}$ using convolution theorem 6

C) Express the function $f(x) = \begin{cases} -e^{kx} & , \quad x < 0 \\ e^{-kx} & , \quad x > 0 \end{cases}$ as Fourier integral .Hence evaluate $\int_0^{\infty} \frac{w.\sin wx}{w^2+k^2}\, dw$ 8