

(Time: 3 hours)

[Total marks: 80]

Note: - (1) Question No. 1 is compulsory.**(2) Answer any three question from Q 2 to Q 6.****(3) Figures to the right indicate full marks.**

- 1 (a) Find the Laplace Transform of $e^{-t} \sin 2t \cos 3t$. 05
- 1 (b) Construct an analytic function whose real part is $y^3 - 3x^2y$ 05
- 1 (c) Find Eigen values of $A^2 - 2A + I$ where $A = \begin{bmatrix} 2 & -1 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & -3 \end{bmatrix}$. 05
- 1 (d) Find Fourier series for $(x) = x^2$ in $0 < x < 2\pi$. 05
- 2 (a) If $\vec{F} = xye^{2z}i + xy^2\cos zj + x^2\cos yk$ find $\text{div}\vec{F}$ and $\text{curl}\vec{F}$ 06
- 2 (b) Find Fourier series of $f(x) = x^3$, $-\pi < x < \pi$. 06
- 2 (c) Find Inverse Laplace Transform of (i) $\frac{2s+3}{s^2+2s+2}$ (ii) $\frac{s+2}{s(s+3)}$. 08
- 3 (a) Find Eigen Values and Eigen Vector of the following matrix 06
- $$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$
- 3 (b) Determine the Constants a, b, c, d if 06
- $$f(z) = x^2 + 2axy + by^2 + i(dx^2 + cxy + y^2) \text{ is analytic}$$
- 3 (c) Find Fourier series for $f(x) = \begin{cases} 1 + 2x/\pi, & -\pi < x < 0 \\ 1 - 2x/\pi, & 0 < x < \pi \end{cases}$ and hence deduce that 08
- $$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots \dots \dots$$
- 4 (a) Prove that $\vec{F} = (y\sin z - \sin x)i + (x\sin z + 2yz)j + (xy\cos z + y^2)k$ is solenoidal and irrotational. 06

4 (b) Evaluate $\int_0^{\infty} e^{2t} t \cos t \, dt$. 06

4 (c) Show that the matrix

$A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ diagonalizable and find transforming matrix and Diagonal matrix. 08

5 (a) Find the inverse Laplace Transform of $\frac{s+4}{(s+1)(s+2)(s+3)}$ by using Partial fraction method. 06

5 (b) Construct an analytic function $f(z) = u + iv$, where $v = (x - y)(x^2 + 4xy + y^2)$. 06

5 (c) i) Show that $\vec{F} = (2xyz^2)i + (x^2z^2 + zcosyz)j + (2x^2yz + ycosyz)k$ is a conservative field. 04

ii) If $\vec{F} = (x + 3y)i + (y - 2z)j + (az + x)k$ is solenoidal, find the value of a 04

6 (a) Find Eigen Values and Eigen Vector of the following matrix 06

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$$

6 (b) Find inverse Laplace transform

i) $\frac{1}{s^2+3s+5}$ ii) $\log \left[\frac{s^2+4}{s+4} \right]$ 06

6 (c) Evaluate $\int_0^{\infty} e^{-2t} \left(\int_0^t e^{-u} u \sin 2u \, du \right) dt$ 08