Time: 3 hour Max. Marks: 80

Note: 1. Question no. **1** is compulsory.
2. Attempt any **three** questions out of remaining **five** questions.
3.Figures to the right indicate full marks.

Q1 (a) Find L $\left[\frac{(\cos at - \cos bt)}{t}\right]$, (05)

(b) Find the constants k, if $f(z) = r^3 cosk\theta + ir^k sin3\theta$ is analytic. (05)

(c) If A = $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ Find A^{50} (05)

(d) If the vector $\vec{F} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$ is is irrotational; find the constants a, b, c. (05)

Q2 (a) Find the analytic function f(z) $in\ terms\ of\ z$ whose real part is $u = sinxcoshy$ (0 6)

(b) Obtain the Fourier series for $f(x) = e^{ax}$ in (0,2π) (0 6)

(c) (i) If $L\{f(t)\} = \frac{1}{s\sqrt{s+1}}$, then find$L\{f(2t)\}$

(ii) Find $L(t^5 \cosh t)$ (08)

Q3 (a) Find L^{-1} $\left[\frac{s}{(s^2+4)(s^2+1)}\right]$ by convolution theorem. (06)

(b) Find Fourier expansion of f(x) = 2x -x^2 in (0,3) (06)

(c) Evaluate by using Green's theorem $\int_C (x^2 - y)dx + (2y^2 + x)dy$, where C is the closed region bounded by $y = 4$ and $y = x^2$ (08)

Q4 (a) If $v = 3x^2y + 6xy - y^3$ show that V is Harmonic function. (06)

(b) Find the Eigenvalues of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ and Show that matrix satisfies the characteristic equation . (06)

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(c) Evaluate (i) $L^{-1}\left\{\frac{1}{s}\tan^{-1}\frac{1}{s}\right\}$ (ii) $L^{-1}\left\{\frac{1}{(S+1)^2+1}\right\}$ (08)

Q5 (a) Obtain the half range Fourier cosine series expansion for

$f(x) = x(2 - x)$ in $(0,2)$. (06)

(b) Find Eigen value and Eigen Vector Of Matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ (06)

(c) Show that $\vec{F} = (y^2 \cos x + z^3)i + (2y \sin x - 4)j + (3xz^2 + 2)k$ is conservative Field. Find (i) Scalar potential for $\vec{F}$ (ii) the work done in moving an object in this field From (0 ,1,-1) to ($\frac{\pi}{2}$, -1 , 2) (08)

Q6 (a) Find the orthogonal trajectory of family of curves given by

$2x - x^3 + 3xy^2 = a$ (06)

(b) Evaluate $\int_0^\infty e^{-3t} t \sin t \, dt$ (06)

(c) Show that the Matrix $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonaisable. Find the diagonal form D And diagonalizing matrix M. (08)
