(3 hours) **Total Marks-80**

N.B. 1) Question No.1 is compulsory.

2) Attempt any THREE questions from Q.No.2 to Q.No.6

3) Figures to the right indicate full marks

Q1 a) Find $L[\frac{\cos 2t \sin t}{e^t}]$ [5]

b) Determine the constants a,b,c,d if $f(z) = x^2 + 2axy + by^2 + i(cx^2 + 2dxy + y^2)$ is analytic. [5]

c) Find Half range cosine series for $f(x) = x(\pi - x), 0 < x < \pi$ [5]

d) Find the directional derivative of $f(x, y, z) = xy^2 + yz^3$ at the point(2,-1,1) in the direction of the vector $i + 2j + 2k$ [5]

Q2) a) Show that the function $u = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic. Find its harmonic conjugate and corresponding analytic function. [6]

b) Find the Fourier series for $f(x) = 1 - x^2$ in (-1,1). [6]

c) Find $i)\ L^{-1}[\frac{e^{-\pi s}}{s^2-2s+2}]$ [8]

$ii)\ L^{-1}[tan^{-1}\left(\frac{s+a}{b}\right)]$

Q3) a) Find the angle between the surfaces $xlogz + 1 - y^2 = 0$, $x^2y + z = 2$ at (1,1,1) [6]

b) Prove that $J'_2(x) = \left(1 - \frac{4}{x^2}\right)J_1(x) + \frac{2}{x}J_0(x)$ [6]

c) Obtain Fourier series for [8]

$$f(x) = \begin{cases} x + \frac{\pi}{2} & , -\pi < x < 0 \\ \frac{\pi}{2} - x & , \quad 0 < x < \pi \end{cases}$$

Hence deduce that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \cdots \ldots$

Q4) a) Using Gauss's Divergence theorem, prove that [6]

$\iint_s (y^2z^2i + z^2x^2j + z^2y^2\,k).\bar{N}ds = \frac{\pi}{12}$ where S is the part of the sphere $x^2 + y^2 + z^2 = 1$ above the xy- plane.

b) Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}.\,cosx$ [6]

c) Solve using Laplace Transform $(D^2 + 2D + 5)y = e^{-t}sint$, when $y(0) = 0, y'(0) = 1$ [8]

Q5) a) Find inverse Laplace Transform using convolution theorem for $\frac{1}{(s-a)(s+a)^2}$ [6]

b) Prove that $J_3(x) + 3J_0(x) + 4J_0'''(x) = 0$ [6]

c) Obtain the complex form of Fourier Series for $f(x) = e^{ax}$ in $(-l, l)$ [8]

Q6) a) Using Green's Theorem in the plane evaluate $\oint(x^2 - y)dx + (2y^2 + x)dy$ around the boundary of the region defined by $y = x^2$, $y = 4$ [6]

b) Show that the map of real axis of the Z-plane is a circle under the transformation $w = \frac{2}{z+i}$. Find its centre and the radius. [6]

c) Find Fourier Integral Representation for [8]

$$f(x) = \begin{cases} 1 - x^2 & for\ |x| \le 1 \\ 0 & for\ |x| > 1 \end{cases}$$
