

(3 Hours)

[Total marks : 80

- Note** : -
- 1) Question number 1 is **compulsory**.
 - 2) Attempt any **three** questions from the remaining **five** questions.
 - 3) **Figures** to the **right** indicate **full marks**.

- Q.1 a) Evaluate $\int_0^{\infty} e^{-2t} \sin^2 2t \, dt$. 05
- b) Find an analytic function $f(z) = u + iv$ where $u + v = e^x(\cos y + \sin y)$. 05
- c) Obtain Fourier series of $x \cos x$ in $(-\pi, \pi)$. 05
- d) Evaluate $\int_C \bar{F} \cdot d\bar{r}$ where $\bar{F} = x^2 i + xy j$ from $(0, 0)$ to $(1, 1)$ along the parabola $y^2 = x$. 05
- Q.2 a) Find half-range cosine series for $f(x) = e^x$, $0 < x < 1$. 06
- b) Prove that $\bar{F} = (x + 2y + az) i + (bx - 3y - z) j + (4x + cy + 2z) k$ is solenoidal and determine the constants a, b, c if $\bar{F}$ is irrotational. 06
- c) Prove that $w = i \left(\frac{z-i}{z+i} \right)$ maps upper half of the z -plane into the interior of the unit circle in the w -plane. 08
- Q. 3 a) Prove that $J_n(x)$ is an even function if n is even integer and is an odd function if n is odd integer. 06
- b) Find the inverse Laplace transform of $\frac{s^2 + 2s + 3}{(s^2 + 2s + 5)(s^2 + 2s + 2)}$. 06
- c) Obtain the complex form of Fourier series for $f(x) = e^{ax}$ in $(0, a)$. 08
- Q. 4 a) Prove that $\nabla f(r) = f'(r) \frac{\bar{r}}{r}$ and hence, find f if $\nabla f = 2r^4 \bar{r}$. 06
- b) Prove that $4J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$. 06

- c)
(i) Find the Laplace transform of $e^{4t} \sin^3 t$. 04
- (ii) Find the Laplace transform of $t \sqrt{1 + \sin t}$. 04
- Q. 5 a) Prove that $\int x \cdot J_{\frac{2}{3}} \left(x^{\frac{3}{2}} \right) dx = -\frac{2}{3} x^{-\frac{1}{2}} J_{-\frac{1}{3}} \left(x^{\frac{3}{2}} \right)$. 06
- b) Find p if $f(z) = r^2 \cos 2\theta + i r^2 \sin p\theta$ is analytic. 06
- c) If $f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2-x), & 1 \leq x \leq 2 \end{cases}$ with period 2, show that 08
- $$f(x) = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \cos(2n+1)\pi x.$$
- Q. 6 a) Show that the set of functions $\cos nx$, $n = 1, 2, 3, \dots$ is orthogonal on $(0, 2\pi)$. 06
- b) Use Stoke's theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$ where 06
- $$\vec{F} = (2x - y) i - yz^2 j - y^2 z k$$
- and S is the surface of hemisphere $x^2 + y^2 + z^2 = a^2$ lying above the xy -plane.
- c) Use Laplace transform to solve 08
- $$\frac{d^2 y}{dt^2} + y = t \text{ with } y(0) = 1, y'(0) = 0.$$
