

Max. Marks: 80

Time: 3 hrs.

N.B. : 1. Q1 is compulsory

2. Attempt any three questions from Q2 to Q6.

3. Figures to the right indicate full marks.

Q1. (a) Evaluate the integral  $\int_C \frac{1}{(z^2+1)(z^2+4)} dz$ ,  $C: |z-2i|=2$ . [5]

(b) A r.v.  $X$  has the distribution [5]

$X:$	0	1	2	3	4	5	6
$p(x):$	$k$	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

Find i)  $k$  ii)  $P(3 < X \leq 6)$

(c) Using Gram Schmidt method, find an orthogonal set of vectors corresponding to  $(1,1,0,1)$ ,  $(-1,0,1,0)$ ,  $(0,0,1,-1)$ . [5]

(d) Find the equations of line of regression of  $y$  on  $x$  for the following data. [5]

$x :$	5	6	7	8	9	10	11
$y :$	11	14	14	15	12	17	16

Q2. (a) Find the Extremal of  $\int_0^1 y y' + (y'')^2 dx$ , [6]

$$y(0)=0, y'(0)=1, y(1)=2, y'(1)=4$$

(b) Find the Laurent series expansion of  $\frac{z+2}{z^2-1}$  convergent in the [6]

domain  $|z| > 1$ .

(c) Reduce the quadratic form  $x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 - 2x_1x_3 + 2x_2x_3$  [8]

to diagonal form by congruent transformation. Obtain the transformation applied in the reduction and Find the rank, index and class value.

Q3. (a) Find the Extremal of  $\int_{x_1}^{x_2} \sqrt{1+(y')^2} dx$ . [6]

(b) There are two brands of smartphone available in the market. A person may buy a smartphone of brand X in 75% and that of Y is 25%. If 95% of brand X and 80% of brand Y perform according to the specification. If the smartphone bought by him is working according to the specification, then what is the probability that it is of brand Y? [6]

(c) Find a singular value decomposition of the matrix  $\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$ . [8]

Q4. (a) Evaluate the integral  $\int_C \frac{\cos^2 z}{z^5} dz$ ,  $C: |z|=1$ , using Cauchy [6]

integral formula.

(b) Find the usual inner product between the two vectors  $(2, 6, 1, -3)$  and  $(3, 2, 1, 0)$ . Find the norm of each vector and verify the Cauchy Schwarz inequality. [6]

(c) The marks of 1000 students of an Engineering college are distributed normally with mean 70 and standard deviation 5. Estimate the number of students whose marks will be i) between 60 and 75 ii) more than 75 [8]

Q5. (a) If C is the rectangle formed by the lines  $x = \pm 2$ ,  $y = \pm \frac{1}{2}$ , then [6]

Evaluate the Integral  $\int_C \frac{2z}{z^4 - 1} dz$

(b) Calculate the rank correlation coefficient for the following data. [6]

x : 10 12 18 16 15 40  
y : 12 18 20 15 50 25

(c) Using Rayleigh-Ritz method, find an approximate solution for the [8]

Extremal of  $\int_0^1 (y')^2 - 4y^2 + 2x^2 y dx$ ,  $y(0)=0$ ,  $y(1)=0$

Q6. (a) Random variables  $X_1$  has mean 5 and variance 5,  $X_2$  has [6]

mean -2 and variance 3,  $X_1$  &  $X_2$  are independent. Find

$E(2X_1 + 3X_2 - 5)$ ,  $\text{Var}(X_1 + X_2)$ ,  $\text{Var}(3X_1 - 2X_2 - 5)$ .

(b) Let  $W_1 = \{(x, y) \mid x, y \in \mathbb{R}, y = mx\}$  and  $W_2 = \{(x, y) \mid x, y \in \mathbb{R}, xy \geq 0\}$ . [6]

Show that  $W_1$  is a subspace and  $W_2$  is not a subspace of  $\mathbb{R}^2$

(c) Fit a second degree parabolic curve to the following data to predict the [8]

annual production where year = 1974 + x .

x	:	0	1	2	3	4	5
Production in crores tons (y)	:	2	4	6	4	3	7

Predict the production (in crores tons) in the year 1980 based on this fitting.