Time: 3 Hours   **Marks:80**

N.B.   1. Question No. 1 is compulsory.
            2. Attempt any THREE out of remaining FIVE questions.

Q. 1 *a)* Find the extremal of the functional $\int_{x1}^{x2}(y^2 + y'^2 - 2y\sin x)\,dx$ (5)

b) Determine whether the set of vectors of the form (a,b,c) where   (5)
b=a+c is subspace of R^3 under usual addition and scalar
multiplication.

c) Find the eigen values of $A^3 - 3A^2 + A$ where   (5)
$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$.

d) Find the value of k and mean if the function   (5)
$$f(x) = \{kx^2(1 - x^3) \quad if \quad 0 \le x \le 1$$
$$0 \qquad\qquad\qquad otherwise\}$$

Q. 2 *a)* Evaluate $\int_c \frac{2z-1}{z(2z+1)(z+2)}\,dz$ where 'c' is the circle $|z| = 1$.   (6)

b) Find the moment generating function of the following   (6)
distribution

x	-2	3	1
P(X=x)	1/3	1/2	1/6

Hence find the first four central moments.

c) Two regression lines are given by 3x +2 y=26 and 6x +y =31 i)   (8)
Find the means   of x and y ii) the correlation coefficient. And
iii) σ_y if $\sigma_x = 3$.

Examine whether the set of real matrices of order 2×2 as

Q 3 *a)* defined by $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ with usual addition and scalar   (6)
multiplication is a vector space.

b) Evaluate$\int_c \frac{sin^6 z}{(z-\frac{\pi}{6})^3}\,dz$ $c: |z| = 1$.   (6)

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c) Show that the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalisable. (8)
Find the transforming matrix and the diagonal matrix.

Q 4 a) Find the curve of given length 'l' which encloses a maximum area. (6)

b) The marks obtained by 1000 students in an examination are found to be normally distributed with mean 70 and standard deviation 5. Estimate the number of students whose marks will be i) between 60 and 75 ii) more than 75. (6)

c) Find all the possible Laurent's expansions of $\frac{z}{(z-1)(z-2)}$ about z= - 2. (8)

Q 5 a) Show that the matrix $A = \begin{bmatrix} 2 & -3 & 3 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ is derogatory and find its minimal polynomial. (6)

b) Find an orthonormal basis for the subspaces of R^3 by applying Gram Schmidt Process where S={(1,2,0),(0,3,1)} (6)

c) Fit a binomial distribution to the following data. (8)

x	0	1	2	3	4	5	6
f	5	18	28	12	7	6	4

Q 6 a) Using Rayleigh Ritz method find an approximate solution for the extremal of the $\int_{x1}^{x2} (2xy + y^2 - y'^2)\, dx$ with y(0)=0 and y(1)=0. (6)

b) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, Show that for $n \geq 3, A^n = A^{n-2} + A^2 - I$. (6)

c) Using Residue theorem evaluate $\int_0^{2\pi} \frac{d\theta}{(2+cos\theta)^2}$. (8)

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