Duration: 3 hours   **Max. Marks 80**

N. B.: 1. **Question No. 1 is Compulsory**.

      2. **Attempt any 3 Questions from Question no. 2 to 6.**

      3. **Figures to the right indicate the full Marks.**

      4. **Statistical tables are allowed.**

Que. 1	a	If λ is an eigen value of square matrix A then prove that $\dfrac{\lvert A \rvert}{\lambda}$ is an eigen value of matrix A^{-1}	**5**
	b	A continuous random variable 'x' has probability density function $f(x) = kx^3 \qquad 0 \le x \le 1$, hence find k, mean and $P(0.3 < x < 0.6)$.	**5**
	c	Find a basis for the orthogonal complement of the subspace in R^3 spanned by the vectors $V_1 = (1, -1, 3)$, $V_2 = (5, -4, -4)$, $V_3 = (7, -6, 2)$	**5**
	d	Evaluate $\displaystyle\int_0^{1+i} z^2 \, dz$ along the curves i. $x^2 = y$ ii. $x = y$	**5**
Que.2.	a	If A=$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ find eigen values and eigen vectors of $A^2 + 2A + I$.	**6**
	b	Find mean and Variance of Binomial distribution	**6**
	c	Find all Taylor and Laurent series expansions for $f(z) = \dfrac{1}{(z-1)(z-2)}$ about z=3 indicating the region of convergence.	**8**
Que.3.	a	Find the curve on which the functional $\displaystyle\int_0^1 (y'^2 + 12 \, xy) \, dx$ with y(0) = 0 and y(1) = 1 is extremal	**6**
	b	Verify Cayley-Hamilton theorem for A=$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^{-1}	**6**
	c	Obtain the equations of the lines of regression for the following data. Also obtain the estimate of X for Y=70.	**8**

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

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Que.4. a By using Cauchy's residue theorem, evaluate $\oint_C \frac{\sin^6 z}{(z-\pi/2)^3}\,dz$ **6**

where C is $|z| = 2$

b Let R^3 have the Euclidean inner product. Using Gram Schmidt process **6**
to transform the basis $\{u_1, u_2\ u_3\}$ into orthonormal basis where

$u_1 = (1,\ 1,\ 1\ ),\ u_2 = (-1,\ 1,\ \ 0),\ \ u_3 = (1,\ 2,\ 1).$

c Determine whether t he matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ is diagonalizable, if yes **8**

diagonalise it.

Que. 5 a Show that the matrix A= $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is derogatory and find the **6**

minimal polynomial of the matrix.

b The weekly wages of 1000 workmen are normally distributed around a **6**
mean of Rs 70 and standard deviation Rs 5. Estimate the number of
workers whose weekly wages will be (i) between 65 and 75 (ii) more
than 80 (iii) estimate the lowest wages of the 100 highest paid workers.

C Solve boundary value problem $y'' + y + x = 0$ **8**

$0 \le x \le 1,\qquad y(0) = y(1) = 1$ by Rayleigh –Ritz method.

Que.6. a If $A = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$ show that $A^{50} = \begin{bmatrix} -149 & -150 \\ 150 & 151 \end{bmatrix}$ **6**

b Between 2 pm and 4 pm, the average number of phone calls per minute **6**
coming into a switchboard of a company is 1.5. Find the probability that
during one particular minute there will be (i) no phone call at all, (ii) at
least 2 calls.

c By using Cauchy residue theorem, evaluate **8**

i. $\int_0^\infty \frac{dx}{(x^2 + 4)(x^2 + 9)}$ ii. $\int_0^{2\pi} \frac{1}{3 + 2\cos\ \theta}\,d\theta$

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