

Max. Marks 80
hours

Duration: 3

- N. B.: 1. Question No. 1 is Compulsory.
2. Attempt any 3 Questions from Question no. 2 to 6.
3. Figures to the right indicate the full Marks.
4. Statistical tables are allowed.

- Que. 1 a If λ is an eigen value of square matrix A then prove that λ^n is an eigen value of square matrix A^n . 5
- b A Continuous random variable X has a probability density function $f(x) = kx^2 e^{-x}$, $x \geq 0$. Find k, mean and variance. 5
- c Find a basis for the orthogonal complement of the subspace in R^3 spanned by the vectors $V_1 = (1, -1, 3)$, $V_2 = (5, -4, -4)$, $V_3 = (7, -6, 2)$ 5
- d Evaluate the complex line Integral $\int_0^{1+i} (x - y + ix^2) dz$ along the straight line from $z=0$ to $z=1+i$ 5
- Que.2. a Find the curve $y=f(x)$ for which $\int_{x_1}^{x_2} y \sqrt{1+y'^2} dx$ is minimum subject to the constraint $\int_{x_1}^{x_2} \sqrt{1+y'^2} dx = l$. 6
- b Seven dice are thrown 729 times . How many times do you expect at least 4 dice to show 3 or 5 ? 6
- c Find all Taylor and Laurent series expansions for $f(z) = \frac{z}{(z-3)(z-4)}$ about $z=1$ indicating the region of convergence. 8
- Que.3. a Find the expectation of (i) the sum (ii) the product of the number of points on the throw of n dice. 6
- b Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find A^{-1} 6
- c Obtain the equations of the lines of regression for the following data. Also obtain the estimate of X for Y=70. 8
- | | | | | | | | | |
|---|----|----|----|----|----|----|----|----|
| X | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 |
| Y | 67 | 68 | 65 | 68 | 72 | 72 | 69 | 71 |

Que.4. a Using Reyleigh-Ritz method, solve the boundary value problem

$$I = \int_0^1 (y'^2 - y^2 - 2xy) dx; \quad 0 \leq x \leq 1. \text{ Given } y(0)=0 \text{ and } y(1)=0$$

6

b Construct an orthonormal basis of R^3 using Gram Schmidt process to $S=\{(3, 1, 4), (-1, 0, 7), (2, 9, 11)\}$

6

c Determine whether the matrix $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$ is diagonalizable, if yes diagonalise it.

8

Que. 5 a Show that the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ is derogatory and find the minimal polynomial of the matrices

6

b Three factories A, B, and C produces 30%, 50% and 20% of the total production of an item. Out of their production 8%, 5% and 1% are defective. Find probability that defective item is produced by factory A

6

c Of a group of men 5% are under 60 inches height and 40% are between 60 and 65 inches. Assuming a normal distribution find the mean height and standard deviation.

8

Que.6. a If $A = \begin{bmatrix} \pi/2 & \pi \\ 0 & 3\pi/2 \end{bmatrix}$, Find $\sin A$

6

b An insurance company found that only 0.01% of the population is involved in a certain type of accident each year. If its 1000 policy holders were randomly selected from the population, what is the probability that not more than two of its clients are involved in such accident next year.

6

c By using Cauchy residue theorem, evaluate

8

i. $\int_0^\infty \frac{dx}{x^2 + 9}$

ii. $\int_0^{2\pi} \frac{1}{5 + 4\cos\theta} d\theta$