(Time : 3 Hours) [Total marks: 80

Note: 1). Question 1 is compulsory.

2) Attempt any 3 questions from Question 2 to Question 6

Q1	Attempt All questions	Marks

A If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$ then find the eigen values for the matrix   5

$$A^3 + 5A + 8I + A^{-1}$$

B Find Laplace transform of $f(t) = te^{-t}\sin(4t)$ 5

C Find the Fourier Series Expansion $f(x) = x$,where $x \in (-\pi, \pi)$ 5

D Determine the constant a,b,c,d if   5
$$f(z) = x^2 + 2axy + by^2 + i(dx^2 + 2cxy + y^2)$$
is analytic.

Q2

A Using Green's theorem in a plane to evaluate the line integral   6

$$\oint_C (xy^2 - y)dx + (x + y^2)dy$$

Where C is the triangle with vertices at (0,0), (2,0) and (2,2) and it is
traversed in anticlockwise direction

B Find the matrix $A_{2\times2}$ whose eigen values are 4 and 1 and their   6

corresponding eigen vectors are $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

C Find the analytic function $f(z) = u + iv$ such that   8
$$u - v = \frac{\cos x + \sin x - e^{-y}}{2\cos x - e^y - e^{-y}} \ when \ f\left(\frac{\pi}{2}\right) = 0$$

Q3

A Find the direction derivative of $\phi(x, y, z) = \sin(xy) + e^{3xz}$ in the   6

direction of the vector $v = \boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}$ at the point $P = \left(1, \frac{\pi}{4}, 1\right)$

B Find an analytic function $f(z)$ whose real part is given   6
$$u(x,y) = x^3 - 3xy^2 + 2x + y$$

C Find the Eigen values and Eigen vectors of   8

$$A = \begin{bmatrix} \dfrac{37}{60} & \dfrac{17}{60} & \dfrac{17}{60} \\ \dfrac{1}{5} & \dfrac{7}{10} & \dfrac{1}{5} \\ \dfrac{1}{12} & -\dfrac{1}{12} & \dfrac{5}{12} \end{bmatrix}$$

And show that it is diagonalizable matrix and find its transforming
matrix and the diagonal form

27200 **Page 1 of 2**

Q4

A Using Stokes theorem to evaluate $\int_C \bar{F} \cdot d\bar{r}$ 6

Where $\bar{F} = (x - y - z)i + (y - z - x)j + (z - x - y)k$ over the paraboloid $x^2 + y^2 = 4 - z$, $z \geq 0$

B Find the orthogonal trajectories of family of curves given by   6
$x^3 y - xy^3 = c$

C Using Convolution theorem, find the inverse Laplace transform of   8
$$\phi(s) = \frac{s + 1}{(s^2 + 2s + 2)(s^2 + 2s + 5)}$$

Q5

A Evaluate $\int_0^\infty \frac{\cos 6t - \cos 4t}{t} \, dt$, using Laplace transforms   6

B Consider the vector field $\bar{F}$ on $\mathbb{R}^3$ defined by   6
$$\bar{F}(x, y, z) = y\,\hat{i} + (z\cos(yz) + x)\,\hat{j} + (y\cos(yz))\,\hat{k}$$
Show that $\bar{F}$ is conservative and find its scalar potential.

C Find the Fourier Series for $f(x)$ in $(0, 2\pi)$ where   8
$$f(x) = \begin{cases} x & , 0 < x \leq \pi \\ 2\pi - x & , \pi \leq x < 2\pi \end{cases}$$
Hence deduce that
$$\sum_{n \in Odd\ natural\ numbers} \frac{1}{n^4} = \frac{\pi^4}{96}$$

Q6

A   6

Obtain half range sine series in $(0, \pi)$ for $f(x) = x(\pi - x)$,
Hence show that
$$\frac{\pi^3}{32} = 1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots$$

B   6

Using Cayley Hamilton theorem find
$$A^6 - 12A^5 + 30A^4 + 72A^3 - 207A^2 - 110A + 330I$$
Where $A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

C   4

i)   Find $L^{-1}\left\{\log\left(\sqrt{\frac{s^2 + a^2}{s^2}}\right)\right\}$

   4

ii)   Find $L^{-1}\left\{\frac{s-1}{(2s+1)^2}\right\}$

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