

Duration: 3 hours

Max. Marks: 80

N.B. (1) Question No. 1 is **COMPULSORY**.

(2) Answer **ANY THREE** questions from Q.2 to Q.6.

(3) Use of Statistical Tables permitted.

(4) Figures to right indicate full marks.

Que. 1 a. Find Laplace Transform of $\sinh 3t. \sin 3t. \cos 5t$ 5

b. Find Fourier series expansion of $f(x) = x^2$ in $(-\pi, \pi)$ 5

c. Find a, b, c, d if $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is an analytic function. 5

d. If $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 4 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ find eigenvalues of $A^2 - 2A + I$ & eigen values of $\text{adj} A$ 5

Que. 2 a. Obtain Fourier series expansion for $f(x) = x |x|$ in $(-1, 1)$ 6

b. By using convolution theorem, find the inverse Laplace transform of $\frac{s}{(s^2+9)(s^2+16)}$ 6

Find the eigenvalues and the eigenvectors of the matrix 8

c. $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

Que. 3 a. Find the analytic function whose real part is $u = e^{2x}(x \cos 2y - y \sin 2y)$ 6

b. Find the Laplace transform of $\sin^5 t$ 6

By using Green theorem, evaluate $\oint_C \bar{F} \cdot d\bar{r}$ where C is the curve enclosing 8

c. the region bounded by $y^2 = 4ax$, $x=a$ in xy plane and $\bar{F} = (2x^2y + 3z^2)i + (x^2 + 4yz)j + (2y^2 + 6xy)k$

Que. 4 a. Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find A^{-1} 6

- b. Find the harmonic conjugate of the function $v = 3x^2y + 6xy - y^3$ and the corresponding analytic function $f(z) = u + iv$ in terms of z 6
- Obtain the half range cosine series of $f(x) = \pi x - x^2$ in $(0, \pi)$, hence 8
- c. deduce that $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$

Que. 5 Find the analytic function $f(z) = u + iv$, in terms of z , if 6

- a. $u + v = \frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2 \cos 2x}$ 6
- b. Show that $\bar{F} = (y^2 \cos x + z^3)i + (2y \sin x - 4)j + (3xz^2 + 2)k$ is conservative. Find its corresponding scalar potential ϕ . 6
- c. Find inverse Laplace transform of the following functions 8
- i. $\tan^{-1}\left(\frac{s}{2}\right)$ ii. $\frac{e^{-3s}}{s^2 - 4s + 5}$

Que. 6 By using stokes theorem, evaluate $\oint_C \bar{F} \cdot d\bar{r}$ where $\bar{F} = x^2i + xyj$ where 'C' is the boundary of the rectangle $x=0, y=0, x=a, y=b$ 6

- b. By using Laplace transform, evaluate, $\int_0^{\infty} \frac{\cos 2t - \cos 3t}{t} dt$ 6

- c. Determine if the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ diagonalizable, hence find its diagonal matrix D and modal matrix 8