Duration : 3 Hours **Marks: 80**

N.B .1) Question No. 1 is compulsory .

2) Attempt any three questions out of the remaining five questions .

3) Figures to the right indicate full marks .

Q.1 **a)** Find the Laplace transform of $f(t) = 2\alpha(1 + te^{-t})^2 . e^{-2t}$ where α is real constant. **5**

b) Find the Fourier series for $f(x) = x$ in $(-3,3)$ **5**

c) In what direction is the directional derivative of $\emptyset(x, y, z) = 2x^2y^2(8z^4)$ at (1,-1,-2) is maximum ? Find its magnitude . **5**

d) Determined the constants A,B,C,D,E & F if $f(z) = (Ax^3 - Bxy^2 + sin6x.cosh6y + C.x) + i(Dyx^2 - 9y^3 + cosExsinhFy + 101y)$is analytic . **5**

Q.2 **a)** Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\ sinx$. **6**

b) Evaluate $\int_0^\infty e^{-8t}\left\{\int_0^t \int_0^t \int_0^t x.sin4x.cos4x.(dx)^3\right\}.dt$ **6**

c) Obtain half range cosine series for $f(x) = x$,$0 < x < 1$ and hence prove that the value of $\frac{\pi^4}{96} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^4}$ using Parseval's identity . **8**

Q.3 **a)** If $\vec{F} = (x + 2y + 2Lz)i + (4Mx - 3y - z)j + (4x + Ny + 2z)k$is irrotational .Find the constants L, M, N .Show that $\vec{F}$ can be expressed as the gradient of the scalar function . **6**

b) Find Fourier series for the following function **6**

$$f(x) = \begin{cases} (x-\pi)^2 & 0 \le x \le \pi \\ 0 & \pi \le x \le 2\pi \end{cases}$$

c) Solve using Laplace transform $(D^2 + 25)y = (K + 6).t$,if $y(0) = 0 , y'(0) = 0$and find the value of the constant K if $y(\pi) = 1$. **8**

Q.4 **a)** Find the translation transformation using cross ratio property, which maps the points $\infty , -1$,1of Z-plane onto the points ∞, 3 ,2 of W-plane . **6**

b) By using Stokes theorem evaluate $\int_C \overrightarrow{F}.\overrightarrow{dr}$ where $\vec{F} = (x^2 + 2y^2)i + (2x^2 - y^2)j$ and C is the boundary of the region enclosed by circle $x^2 + y^2 = 9$, $x^2 + y^2 = 36$. **6**

c) Find Inverse Laplace transform of **8**
$i)\left\{\frac{s+\alpha}{s^2+16}\right\}$ $and\ find\ the\ constant\ \alpha\ , if\ f\left(\frac{\pi}{8}\right) = 1$ $ii)$ $\left\{\frac{s+6}{(s-5)^2+121}\right\}$

Q.5 **a)** Define Orthogonal set of functions on (a,b) . If $f(x) = P_1 f_1(x) + P_2 f_2(x) + P_3 f_3(x)$, where P_1, P_2 , P_3 are constants and $f_1(x), f_2(x), f_3(x)$are orthogonal functions on (a,b) ,Then show that $\int_a^b [f(x)]^2 . dx = {P_1}^2 R_1 + {P_2}^2 R_2 + {P_3}^2 R_3$ where R_i' are non zero for $i = 1,2,3$. **6**

b) Find the analytic function $f(z) = u + iv$ in terms of Z if $u - 3v = x^2 - y^2 - 5x + y + 2$. **6**

c) Verify Green's theorem for $\int_C (x^2)dx - (xy)dy$, C is a triangle whose vertices are A(0,2) , B(2,0) , C(4,2) in the XY-plane . **8**

Q.6 **a)** Find the image of the real axis of the Z-plane under the transformation $w = \frac{1}{z+i}$ onto the W-plane . **6**

b) Find Laplace transform of $f(t) = e^{-4t}.cos4t.sin4t.H(t - \frac{\pi}{2})$ **6**

c) Obtain Complex form of Fourier series for $f(x) = cosh2x + sinh2x$ in $(-2,2)$. **8**
