

Duration – 3 Hours

Total Marks : 80

(1) N.B.:–Question no 1 is compulsory.

(2) Attempt any THREE questions out of remaining FIVE questions.

Q.1) a) Find Laplace Transform of the periodic function $|\sin t|$. (5)b) Find the half range sine series of $f(x) = lx - x^2$, in $(0, l)$. (5)c) Find the directional derivative of $x^3 + y^3 + z^3 - xyz$ at $P(1,1,1)$ in the direction normal to the surface $x \log z + y^2 = 4$ at $Q(1, -2, 1)$. (5)d) Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$. (5)Q.2) a) Show that the image of the rectangular hyperbola $x^2 - y^2 = 1$ under the transformation $w = \frac{1}{z}$ is the lemniscates $\rho^2 = \cos 2\phi$. (6)b) Evaluate $\int_0^\infty e^{-t} \left(\int_0^t u^4 \sinh u \cosh u \, du \right) dt$. (6)c) Obtain Fourier series of $f(x) = \begin{cases} 1 + (2x/\pi) & -\pi \leq x \leq 0 \\ 1 - (2x/\pi) & 0 \leq x \leq \pi \end{cases}$. Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$. (8)Q.3) a) Show that the set of functions $\left\{ \sin\left(\frac{\pi x}{2L}\right), \sin\left(\frac{3\pi x}{2L}\right), \sin\left(\frac{5\pi x}{2L}\right), \dots \right\}$ forms an Orthogonal set over the interval $[0, L]$. Construct corresponding orthonormal set. (6)b) Prove that $\nabla \times \left[\frac{\vec{a} \times \vec{r}}{r^3} \right] = \frac{-\vec{a}}{r^3} + \frac{3(\vec{a} \cdot \vec{r})\vec{r}}{r^5}$. (6)c) Find the analytic function $f(z) = u + iv$, if $u + v = \frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2 \cos 2x}$. (8)Q. 4) a) Verify Green's theorem for $\int_C (xy + y^2)dx + x^2 dy$ where C is the closed path formed by $y = x, y = x^2$. (6)b) Prove that $\int J_5(x) dx = -J_4 - \frac{4}{x} J_3(x) - \frac{8}{x^2} J_2(x)$. (6)

- c) Solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 3te^{-t}$, $y(0) = 4$, $y'(0) = 2$ using Laplace Transform. (8)

- Q. 5 a) Find the Bilinear Transformation which maps $z = 1, i, -1$ onto the points $w = i, 0, -i$. (6)

- b) Find Inverse Laplace Transform of $\frac{(s+2)^2}{(s^2+4s+8)^2}$ using Convolution theorem. (6)

- c) Evaluate $\iint_S \vec{F} \cdot \vec{n} dS$ where S is the surface of the cube bounded by $x=0, y=0, z=0, x=1, y=1, z=1$ and $\vec{F} = 4xzi - y^2j + yzk$. (8)

- Q. 6 a) Evaluate $\int_C (x+2y)dx + (x-z)dy + (y-z)dz$ where C is the boundary of the triangle with vertices $(2,0,0), (0,3,0), (0,0,6)$ oriented in the anti-clockwise direction. (6)

- b) Find the Fourier integral representation for $f(x) = \begin{cases} 1-x^2 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ (6)

- c) Evaluate the inverse Laplace transformation of (8)

a) $\log\left(\frac{s^2+a^2}{(s+b)^2}\right)$ b) $\frac{e^{-2s}}{s^2+8s+25}$
