QP CODE: 40558

(3 hours) **Max. Marks: 80**

N.B. (1) Question No. 1 is compulsory.
(2) Answer any three questions from Q.2 to Q.6.
(3) Use of Statistical Tables permitted.
(4) Figures to the right indicate full marks.

Q.1 **(a)** Find all the basic solutions to the following problem:
Maximize $z = x_1 + x_2 + 3x_3$ subject to

$$x_1 + 2x_2 + 3x_3 = 9$$
$$3x_1 + 2x_2 + 2x_3 = 15$$
$$x_1, x_2, x_3 \geq 0$$ **05**

(b) Evaluate $\oint z\, dz\ from\ z = 0\ to\ z = 1 + i\ along\ the\ curve\ z = t^2 + it.$ **05**

(c) A sample of 100 students is taken from a large population. The mean height of the students in this sample is 160 cm. Can it be reasonably regarded that in the population, the mean height is 165 cm, and the standard deviation is 10 cm? **05**

(d) The sum of the Eigen values of a 3 × 3 matrix is 6 and the product of the Eigen values is also 6. If one of the Eigen value is one, find the other two Eigen values. **05**

Q.2 **(a)** Evaluate $\oint \frac{\sin^6 z}{(z-\pi/6)^n} dz$ where c is the circle $|z| = 1$ for n = 1, n =3 **06**

(b) The following data is collected on two characters. Based on this, can you say that there is no relation between smoking and literacy? Use Chi-square test at 5% Level of significance. **06**

	Smokers	Non-smokers
Literates	83	57
Illiterates	45	68

(c) Solve the following LPP using Simplex Method
Maximize $z = 3x_1 + 5x_2$
subject to
$3x_1 + 2x_2 \leq 18,$

$$x_1 \leq 4,$$
$$x_2 \leq 6,$$
$$x_1, x_2 \geq 0$$ **08**

Q.3 **(a)** Find the Eigen values and Eigen vectors of the following matrix.

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$$ **06**

(b) The incomes of a group of 10,000 persons were found to be normally distributed with mean of Rs. 750 and Standard deviation of Rs. 50. What is the lowest income of richest 250 ? **06**

(c) Expand $\frac{z^2-1}{z^2+5z+6}$ around z = 0. **08**

[Turn over

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Q.4 **(a)** The mean breaking strength of cables supplied by a manufacturer is 1800 with S.D 100. By a new technique in the manufacturing process it is claimed that the breaking strength of the cable has increased. In order to test the claim a sample of 50 cables are tested. It is found that the mean breaking strength is 1850. Can we support the claim at 1% LOS. **06**

(b) Using the Residue theorem, Evaluate $\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$ **06**

(c) (i) Out of 1000 families with 4 children each, how many would you expect to have (I) at least one boy, (II) at most 2 girls.
(ii) Find the Moment Generating Function of Poisson distribution and hence find its mean. **04+04**

Q.5 **(a)** Check whether the following matrix is Derogatory or Non-Derogatory: **06**

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$$

(b) The means of two random samples of sizes 9 and 7 are 196 and 199 respectively. The sum of the squares of the deviations from the mean is 27 and 19 respectively. Can the samples be regarded to have been drawn from the same normal population? **06**

(c) Use the dual simplex method to solve the following L.P.P. **08**
Minimise $z = 6x_1 + 3x_2 + 4x_3$
subject to

$$x_1 + 6x_2 + x_3 = 10$$
$$2x_1 + 3x_2 + x_3 = 15$$
$$x_1, x_2, x_3 \geq 0$$

Q.6 **(a)** Show that the matrix A satisfies Cayley-Hamilton theorem and hence find A^{-1} . **06**

Where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$

(b) The Probability Distribution of a random variable X is given by **06**

X :	-2	-1	0	1	2	3
P(X = x):	0.1	k	0.2	2k	0.3	k

Find k, mean and variance.

(c) Using Kuhn-Tucker conditions, solve the following NLPP **08**
Maximize $z = x_1^2 + x_2^2$
subject to

$$x_1 + x_2 - 4 \leq 0$$
$$2x_1 + x_2 - 5 \leq 0$$
$$x_1, x_2 \geq 0$$