Time:3Hours **Max. Marks: 80**

N.B.
1) **Q.1 is compulsory.**
2) Solve any 3 questions out of remaining 5 questions.
3) Assumptions made should be clearly stated.
4) Draw the figures wherever required.

Q.1 Solve any four of the following questions.

a) Check if $(p\rightarrow q)\rightarrow[(\sim p\rightarrow q)\rightarrow q]$ is a tautology? **5**

b) Draw the Hasse diagram for [{2,4,5,8,10,12,20,25}, /]. Is it a Poset? **5**

c) Define Eulerian and Hamiltonian Graph. Give examples of following type of graph **5**
i) Eulerian but not Hamiltonian
ii) Not Eulerian but Hamiltonan

d) Explain types of Quantifiers . Represent the following sentences using Quantifiers **5**
i) All hardworking students are clever.
ii) There is a student who can speak Hindi but does not know Marathi

e) State the Pigeonhole principle and prove that in any set of 29 persons at least five persons must have been born on the same day of the week **5**

Q.2

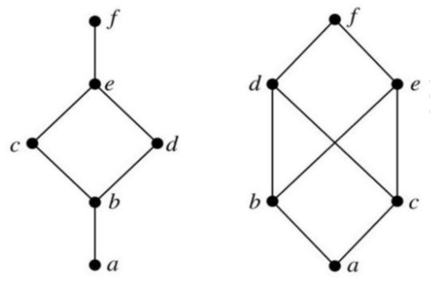
a) Show that the set of all positive rational numbers forms an abelian group under the composition * defined by a*b=(ab)/2 **10**

a) What is a transitive closure? Explain Warshall's algorithm for finding transitive closure with an example. **10**

Q.3

a) By using mathematical induction, prove that the given equation is true for all positive integers. **6**
$1 \times 2 + 3 \times 4 + 5 \times 6 + \ldots. + (2n - 1) \times 2n = n(n+1)(4n-1)/3$

b) Define Lattice? Which of the following is lattice? **8**



(a) (b)

c) Determine the sequence of which recurrence relation is $a_n = 2a_{n-1} - a_{n-2}$ with initial conditions $a_1 = 1.5$, $a_2 = 3$. **6**

Q.4

a) Let A={1, 3,6,9,15,18,21} & R be the relation of divisibility. **8**

i) Write the pairs in a relation set R.

ii) Construct the Hasse diagram.

iii) What are the Maximal and Minimal elements?

iv) Is this poset a distributive lattice? Justify your answer.

b) **6**

Let $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Determine (3, 6) group code $e_H : B^3 \to B^6$

c) Write a short note on Types of Graphs. **6**

Q.5

a) Let(Z,*) be an algebraic structure, where Z is set of integers and the operation * is defined by a*b= maximum of (a,b). Is (Z,*) a Semigroup? Is (z,*) a Monoid? Justify your answer. **8**

b) Define the term Surjective function. Let E be the set of all even numbers then f: N-->E:f(x)=2x , check if it is surjective,bijective?Justify your answer. **4**

c) Give the examples of relation R on S = {a, b, c,d} having stated property. **8**

i) R is an equivalence relation.

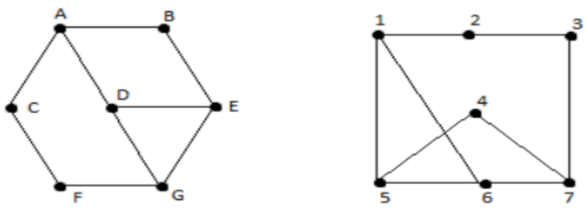
ii) R is symmetric but not transitive

iii) R is both symmetric and antisymmetric

iv) R is neither symmetric nor antisymmetric.

Q.6

a) Define Isomorphic graphs and check whether the following graphs are Isomorphic ? **8**



b)In a group(G,*) , Prove that the inverse of any element is unique and identity element is also unique. **6**

c) Define Relation. Let **6**

f: R→R is defined as $f(x) = x^2$

g: R→R is defined as $g(x) = 3x^2+1$

h: R→R is defined as $h(x) = 9x-2$

find (hof)og, go(foh).

****