(Time: 3 hours)　　　　　　**Max. Marks: 80**

N.B.　(1) Question No. 1 is compulsory.

　　　(2) Answer any three questions from Q.2 to Q.6.

　　　(3) Use of Statistical Tables permitted.

　　　(4) Figures to the right indicate full marks

Q1.

(a)　Find the Laplace transform of　　$t \sqrt{1 + \sin t}$　　　　　　　　　　　[5]

(b)　Find the constants a, b ,c, d, e if　　　　　　　　　　　　　　　　[5]

　f(z) = ($ax^3 + b x y^2 + 3 x^2 + c y^2 + x$) $+ i (d x^2 y - 2 y^3 + e x y + y$) is analytic.

(c)　Calculate the Spearman's rank correlation coefficient R　　　　　　　[5]

　X　:　85 ,　74 ,　85 ,　50,　65,　78,　74,　60,　74,　90

　Y　:　78　91, 78,　58,　60,　72,　80,　55, 68,　70

(d)　Find inverse Laplace transform of　　$tan^{-1} \left(\frac{s+a}{b} \right)$.　　　　　　[5]

　Q2.

(a)　Find the Laplace transform of　　$e^{-4t} \int_0^t u \sin 3u \, du$　　　　　　　[6]

　(b)　find the value of k if the function　　　f (x) = k x^2 ($1 - x^3$) ,　　　$0 \leq x \leq 1$.

　　　　　　　　　　　　　　F (x) = 0　　　　　　　　otherwise

　Is a probability density function. find mean and variance.　　　　　　　[6]

(c)　Obtain the Fourier series to represent f (x) = x^2 in (0, 2π)

　　　Hence show that　$\frac{\pi^2}{12}$ = $\frac{1}{1^2}$ - $\frac{1}{2^2}$ + $\frac{1}{3^2}$ - $\frac{1}{4^2}$ …….　　　　　[8]

Q3.

(a)　Find the analytic function f (z) = u + iv such that　　　　　　　　[6]

u +v = $\frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2 \cos 2x}$.

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(b) Using convolution theorem Find inverse Laplace transform of $\frac{s^2}{(s^2+9)(s^2+4)}$. [6]

(c) Fit a second-degree parabolic curve to the following data

Year (x)   : 1974   1975   1976   1977   1978   1979   1980   1981

Production (y) : 12   14   26   42   40   50   52   53.   [8]

Q4.

(a)   Obtain the   Fourier series to represent f (x) = 9 - x^2 in (-3, 3).   [6]

(b) . Find the coefficients of regression and hence obtain the equation of the lines of Regression for the following data

X :   78 ,   36 , 98 , 25 , 75, 82, 90, 62, 65, 39 .

Y :   84 , 51,   91, 60,   68, 62 , 86, 58, 53, 47.   [6]

(c) Prove that $\int_0^\infty e^{-t} \frac{\sin 2t + \sin 3t}{t}$ dt = $\frac{3\pi}{4}$.   [8]

Q5.

(a) Find the orthogonal trajectories of the family of curves   3 x^2y + 2x^2 - y^3- 2 y^2 = c. [6]

(b)   If X denotes the outcome when a fair die is tossed, find Moment generating function

Of X and hence find the mean and variance of X.   [6]

(c)   Obtain the half range cosine series of   f (x) = x ($\pi - x$) in (0, π)

Hence show that $\frac{\pi^4}{96}$ = $\frac{1}{1^4}$ + $\frac{1}{2^4}$ + $\frac{1}{3^4}$ + $\frac{1}{4^4}$ ……..   [8]

Q6.(a) Find inverse Laplace transform of $\frac{s+29}{(s + 4)(s^2+9)}$.   [6]

(b)   The probability density function of a random variable X is

   X   : 0   1   2   3   4   5   6

  P (X =x) : k   3k   5k   7k   9k   11k   13k

  Find k , p (X < 4) , P (3 < X ≤ 6) .   [6]

(c)   Verify Laplace equation for u = $\left(r + \frac{a^2}{r} \right)$ cos θ.also find v and f (z) .   [8]

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